ADHESIVE BONDED-JOINTS ASSEMBLIES ANALYSIS

Ovidiu V. Nemeş*, Frédéric A. Lachaud, Abdelkader K. Mojtabi*****
***TU of Cluj-Napoca, Romania, **ENSICA Toulouse, France, ***UPS Toulouse, France**

Keywords: *adhesive joints, glass fibers, modeling, finite element analysis, stress transfer*

Abstract

This work presents a theoretical model of cylindrical assemblies joined with adhesive, based on an energy method. After the determination of the cinematically acceptable field of stresses, according to the applied load, a variational calculus on the expression of elastic potential energy leads to the complete expression of the stress field in the whole assembly. A first parametric analysis (geometrical and physical parameters) is carried out on an assembly of tubes and makes it possible to determine the stress variations and deduce the optimal length and the thickness of the adhesive. The model is validated by comparison with a finite elements method. For the assembly, the total force-displacement behaviour is well defined. Thus the analytical model makes it possible to determine the rigidity of the assembly and to obtain a simple formulation very rapidly which gives the total behaviour of the assembly.

1 Introduction

The mechanical performance of an adhesive bonded joint is related to the distribution of the stresses in the adhesive layer. The first studies are developed on plane assemblies with simple covering submitted to traction. The work of Volkersen [1] developed into 1944 leads to a false evaluation of the level of maximum constraint because the effects of bending of the supports are not taken into account. From the first works of Volkersen [1] which give only a distribution of the shear stress in the adhesive joint to the more recent studies by finite elements, many formulations have made it possible to define the field of stresses in such assemblies better and better.

Lubkin and Reissner [2] present an analysis of stresses in tubular assemblies subjected to an axial loading and give a solution of the peeling stress distribution in the thickness of the adhesive. The tubes being supposed of small thickness, they use the theory of thin walls to build the stress field. Their analysis assumes that the work of shear and peel stresses in the two tubes is negligible relative to that of the same stresses in the adhesive.

Alwar and Nagaraja [3] made a study by finite elements of tubular joining subjected to an axial load taking into account the viscoelastic behaviour of the adhesive. They also show that the viscoelastic behaviour of the adhesive makes it possible to predict a considerable reduction in the maximum stresses at the ends of the joint.

Shi and Cheng [4] they built a first stress field using the equilibrium equations and the conditions of continuity of the stresses at the interfaces using an equation of compatibility. They then calculate the potential energy associated with this field and using the theorem of minimal complementary energy, they obtain a system of differential equations, the solutions of which are used to determine the optimal field.

By minimizing the potential energy associated to the stress field using variational methods and some simplifying assumptions Nemes, Lachaud and Mojtabi [5], [6] has developed new analytical models to a fast pre dimensioning of the adhesive cylindrical bonded assemblies.

2 Analytical models

All work has encountered difficulties in modeling the stress field in the vicinity of the ends of the joint. The method used to obtain the optimal field consists of: Construction of a statically acceptable field, Calculation of the potential energy associated with the stress field, Minimization of this energy by variational method, Resolution of the differential equation obtained.

2.1 Stress field definition

In this work we consider an assembly of bonded tubes.

This assembly is subjected to a tensile load whose geometrical definitions are given in figure 1.

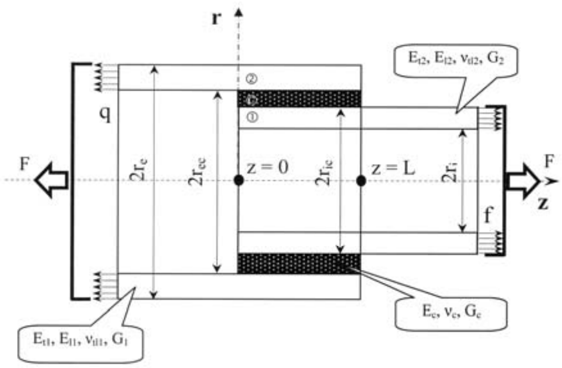


Fig.1. Cylindrical assemblies

The parameters of the assembly are:

- E_{it}, E_{il} - Transversal and longitudinal Young's modulus,
- ν_{tli} - Poisson's ratio,
- r_i - Internal radius of the inner tube,
- r_{ic} - External radius of the inner tube,
- r_{ec} - Internal radius of the external tube,
- r_e - External radius of the external tube,
- L - Joining length,
- f - Tensile stress along z axis on the inner tube,
- q - Tensile stress along z axis on the external tube.

The stresses in various materials will be located by the index (i), (i = ① for the inner tube, © for the adhesive and ② for the outer tube).

To build the statically acceptable field, we will adopt the following hypotheses:

- the symmetry of revolution imposes that the shear stresses are null:

$$\tau_{r\theta} = \tau_{z\theta} = 0 \qquad (1)$$

- the normal stress in the adhesive will be neglected:

$$\sigma_{zz}^{(©)} = 0 \qquad (2)$$

- the axial stress will be a function only of variable z.

The stress field is thus reduced to:

- for the inner tube (①):

$$\sigma_{zz}^{(1)}(z),\ \tau_{rz}^{(1)}(r,z),\ \sigma_{\theta\theta}^{(1)}(r,z),\ \sigma_{rr}^{(1)}(r) \qquad (3)$$

- for the inner tube (©):

$$\sigma_{\theta\theta}^{(©)}(z),\ \tau_{rz}^{(©)}(r,z),\ \sigma_{rr}^{(©)} = cst. \qquad (4)$$

- for the inner tube (②):

$$\sigma_{zz}^{(2)}(z),\ \tau_{rz}^{(2)}(r,z),\ \sigma_{\theta\theta}^{(2)}(r,z),\ \sigma_{rr}^{(2)}(r) \qquad (5)$$

We take an elementary volume of adhesive bonded joint of length dz.

We express all the stress fields' components function of the normal stress in the inner tube ($\sigma_{zz}^{(1)}$) like follow:

$$\tau_{rz}^{(1)}(r,z) = \frac{\left(r_i^2 - r^2\right)}{2r}\frac{d\sigma_{zz}^{(1)}}{dz} \qquad (6)$$

$$\sigma_{\theta\theta}^{(1)}(r,z) = \frac{r_i^2 - r^2}{2}\frac{d^2\sigma_{zz}^{(1)}}{dz^2} + \alpha_1[2r - r_i] \qquad (7)$$

$$\sigma_{zz}^{(©)} = 0 \qquad (8)$$

$$\tau_{rz}^{(c)}(r,z) = \frac{\left(r_i^2 - r_{ic}^2\right)}{2r}\frac{d\sigma_{zz}^{(1)}}{dz} \qquad (9)$$

$$\sigma_{\theta\theta}^{(c)}(z) = \frac{r_i^2 - r_{ic}^2}{2}\frac{d^2\sigma_{zz}^{(1)}}{dz^2} + \alpha_1[r_{ic} - r_i] \qquad (10)$$

$$\sigma_{zz}^{(2)}(r,z) = \left(\frac{r_{ic}^2 - r_i^2}{r_e^2 - r_{ec}^2}\right)\left(f - \sigma_{zz}^{(1)}\right) \qquad (11)$$

$$\tau_{rz}^{(2)}(r,z) = \frac{\left(r_e^2 - r^2\right)\left(r_{ic}^2 - r_i^2\right)}{2r(r_{ec}^2 - r_e^2)}\frac{d\sigma_{zz}^{(1)}}{dz} \qquad (12)$$

$$\sigma_{\theta\theta}^{(2)}(r,z) = \frac{(r_e^2 - r^2)(r_{ic}^2 - r_i^2)}{2(r_{ec}^2 - r_e^2)}\frac{d^2\sigma_{zz}^{(1)}}{dz^2} + \\ + \alpha_1\underbrace{\frac{r_{ic} - r_i}{r_{ec} - r_e}}_{\alpha_2}[2r - r_e] \qquad (13)$$

2.2 Deformation energy calculation

We have express the deformation energy like follow:

$$\xi_P = \pi \int_0^L \int_{r_i}^{r_{ic}} \left[\frac{\sigma_{\theta\theta}^{(1)2}}{E_{1t}} + \frac{\sigma_{zz}^{(1)2}}{E_{1l}} - \right.$$
$$\left. - \frac{2\nu_{tl1}}{E_{1t}} \sigma_{zz}^{(1)} \sigma_{\theta\theta}^{(1)} + \frac{\tau_{rz}^{(1)2}}{G_1} \right] rdrdz +$$
$$+ \pi \int_0^L \int_{r_{ic}}^{r_{ec}} \left[\frac{\sigma_{\theta\theta}^{(c)2}}{E_c} + \frac{2(1+\nu_c)}{E_c} \tau_{rz}^{(c)2} \right] rdrdz + \qquad (14)$$
$$+ \pi \int_0^L \int_{r_{ec}}^{r_e} \left[\frac{\sigma_{\theta\theta}^{(2)2}}{E_{2t}} + \frac{\sigma_{zz}^{(1)2}}{E_{2l}} - \right.$$
$$\left. - \frac{2\nu_{tl2}}{E_{2t}} \sigma_{zz}^{(2)} \sigma_{\theta\theta}^{(2)} + \frac{\tau_{rz}^{(2)2}}{G_2} \right] rdrdz$$

We can simplify the energy's expression function $\sigma_{zz}^{(1)}$:

$$\xi_P = \pi \int_0^L \left[A\sigma_{zz}^{(1)2} + B\sigma_{zz}^{(1)} \frac{d^2\sigma_{zz}^{(1)}}{dz^2} + C\left(\frac{d\sigma_{zz}^{(1)}}{dz}\right)^2 + \right.$$
$$\left. + \tilde{D}\sigma_{zz}^{(1)} + E\left(\frac{d^2\sigma_{zz}^{(1)}}{dz^2}\right)^2 + \tilde{F}\frac{d^2\sigma_{zz}^{(1)}}{dz^2} + \tilde{K} \right] dz \qquad (15)$$

Where:

$$\tilde{D} = D + \alpha_1 k, \qquad (16)$$
$$\tilde{F} = F + \alpha_1 h,$$
$$\tilde{K} = K + \alpha_1^2 m$$

The A, B, C, D, E, F, K constants and k, h, m depend on the load and on the dimensional and mechanical specifications of the two tubes and adhesive [5]

2.3 Parametric study

After the model expression we have made some parametric studies concerning the influence of the adhesive overlap length, the assembly rigidity, the relative rigidity, adhesive thickness.

We observe:

- By increasing the overlap length gradually we have the reduction of shear stress in the medium of the joint and the displacement of the peaks of stresses towards the free edges,
- The maximum peaks increase slightly when the elastic modulus increases,
- the maximum peaks on the two edges are no longer equal if the ratio E_2/E_1 is different from 1,
- As the thickness of adhesive increases, the values of the stresses decrease at the free edges.

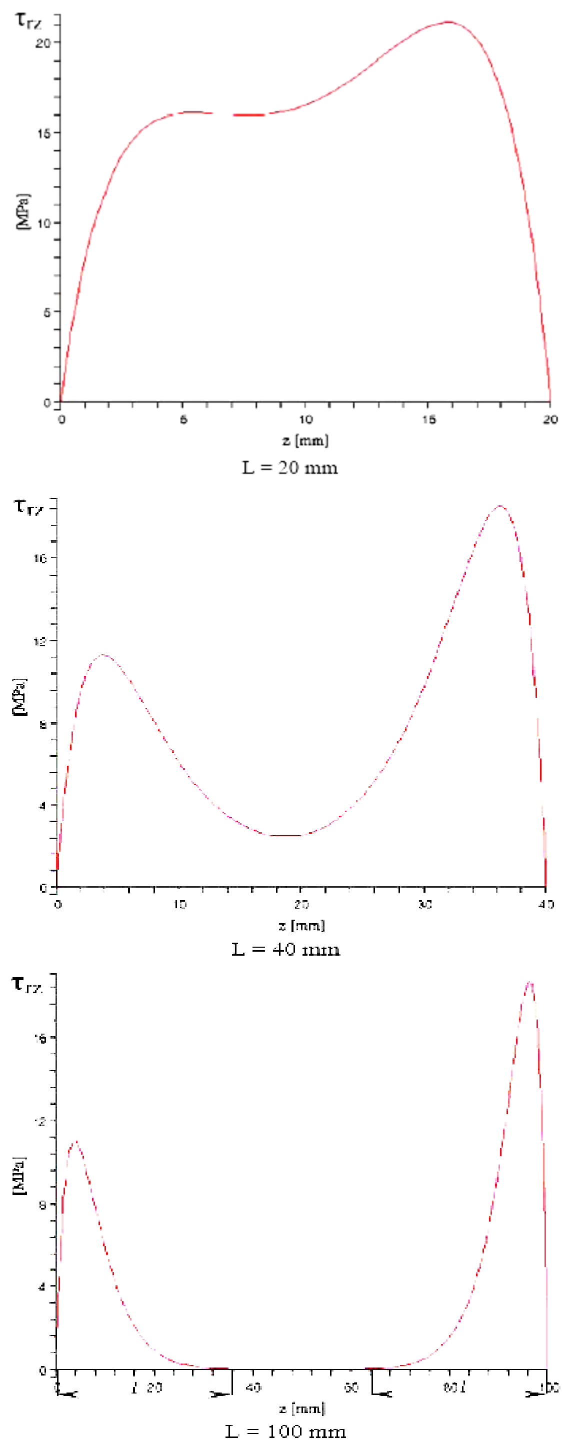


Fig. 2. Shear stress (τ_{rz}) distribution according with the covering length.

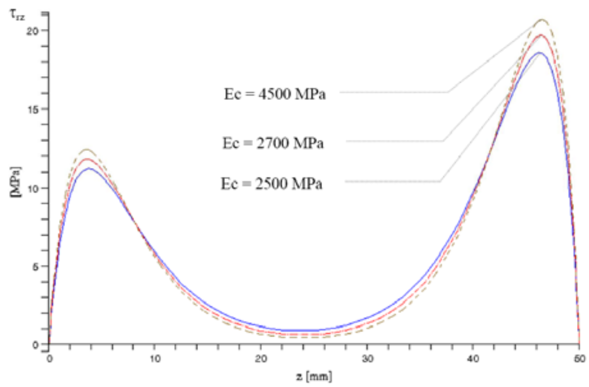


Fig. 3. Shear stress (τ_{rz}) variation according to Young's modulus of the adhesive.

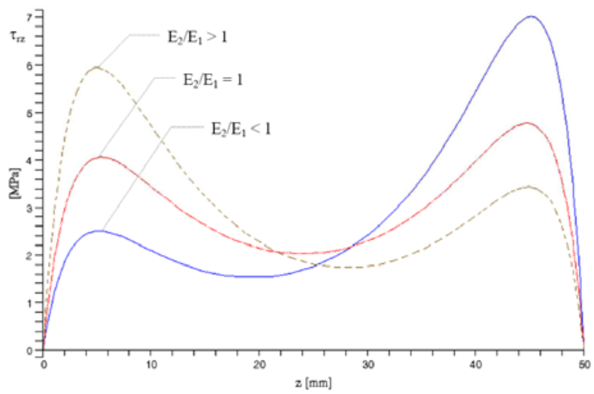


Fig.4. Shear stress (τ_{rz}) variation according to relative rigidity.

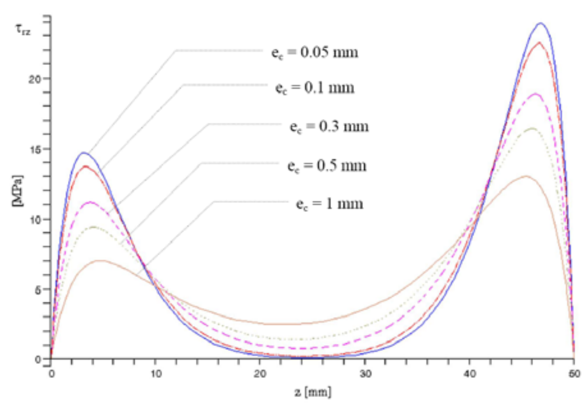


Fig.5. Shear stress (τ_{rz}) variation according to adhesive thickness.

3 Numerical analysis

The objective of this study is to compare our analytical models of the adhesive-bonded joints with models made of finite elements.

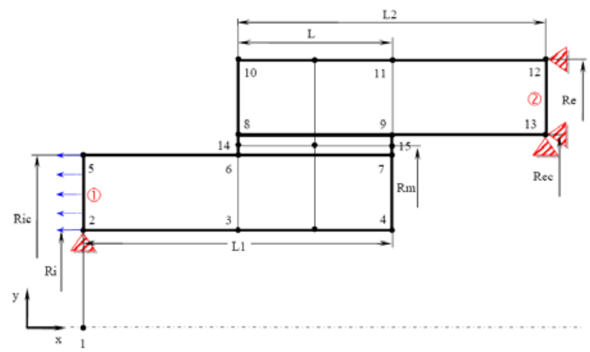


Fig. 6. CAD diagram of a cylindrical adhesive bonded joint.

The C.A.D. diagram, basis of the finite element model, is presented in figure 2. The diagram also describes the boundary conditions and the loading applied. The cylindrical assembly is modelled by 2D quadrangles of degree 2 finite elements with the axis symmetric assumption ($x \rightarrow z$, $y \rightarrow r$, $z \rightarrow \theta$). The displacements along x and y in face ② of the external tube and those along y in face ① of the internal tube are blocked. The load is applied as a pressure on face ① (Figure 6).

Figure 7 shows an example of the grid used in this study where all the finite elements are quadrangles. We imposed ten finite elements according to the adhesive.

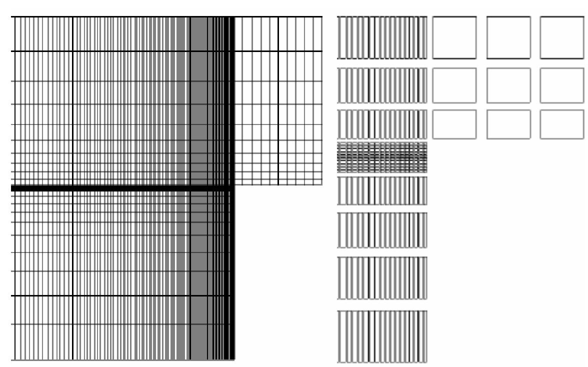
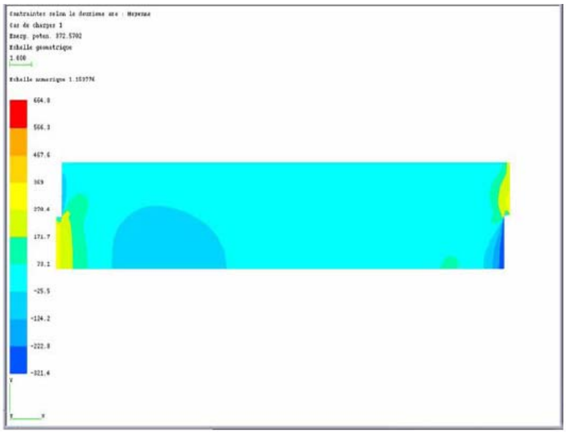
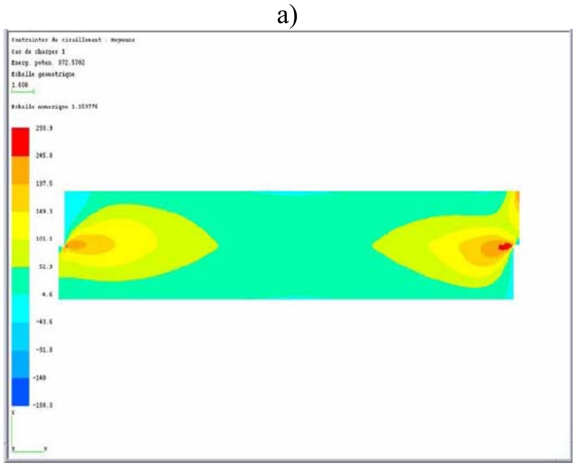


Fig. 7. Numerical modelling of a cylindrical bonded joint with quadrangles elements.

Figure 8 shows the stress distributions in the assembly in the form of cartography.



a)



b)

Fig.8. Stress distribution: a) orthoradial stress ($\sigma_{\theta\theta}$); b) shear stress (τ_{xy}).

3.1 Load transfer

To compare the analytical model with the finite elements model we determine the load transfer in the middle of the bonded substrates, figure 9, in a cylindrical metal-composite assembly.

We can note some differences in the variation of the load given by the ideal model in the case of a metal-composite assembly, (Figure 9), while it still has a similar evolution.

The point of equivalence in stress in the two substrates is shifted (according to the length of the join) from approximately 15% in the finite element analysis. It should be noted that the position of this point varies according to the characteristics of the substrates: it is centered compared to the length of the join for substrates of equivalent total rigidities, and shifts on both sides as a function of the ratio of the rigidities of the bonded substrates.

The ideal analytical model has the same aspect as the finite element model (FEM).

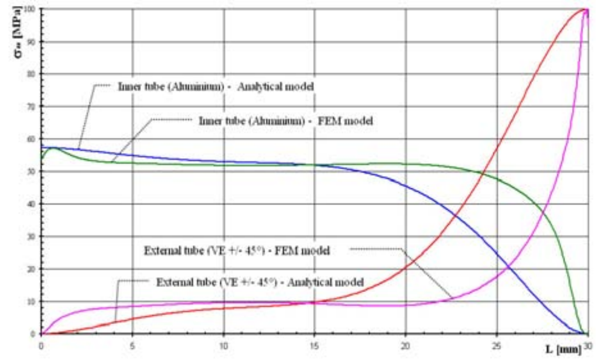


Fig. 9. Load transfer in an AU 2024 T3 - AV 119 - VE ±45° assembly.

3.2 Stress analysis in the adhesive

The stresses in the adhesive make it possible to predict the failure of the adhesive-bonded joint. Their distribution is thus of primary importance on the prediction of this force.

Figure 10 and 11 presents the distribution of the orthoradial and shear stresses, according to the covering length. In this case, the stresses given by the ideal model are similar to those given by finite elements. The greatest differences (30%) are seen on the maximum amplitudes where the ideal model underestimates these values: the edge effect due to a local inflection of the substrates is not taken into account.

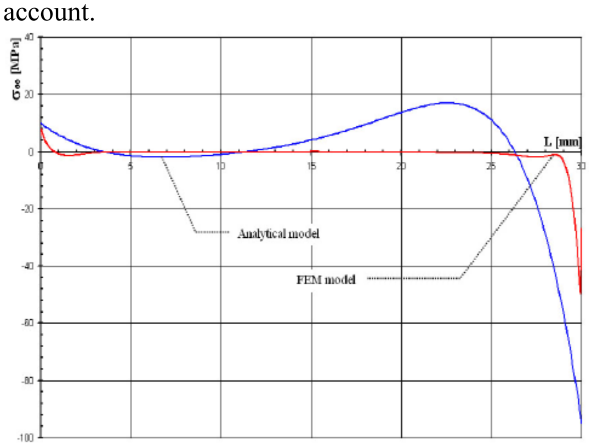


Fig. 10. Stress distributions in the adhesive layer of a cylindrical assembly AU 2024 T3 - AV 119 - VE±45°: The orthoradial stress ($\sigma_{\theta\theta}$).

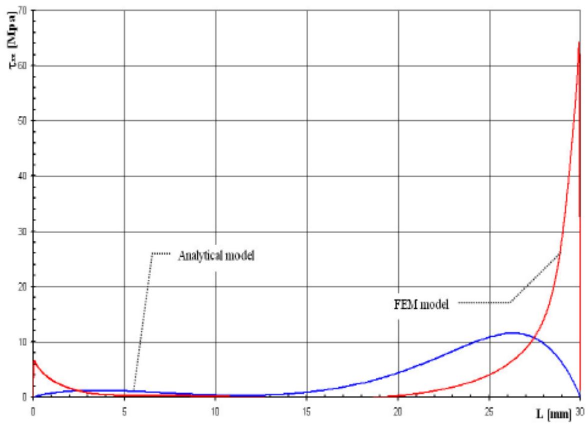


Fig. 11. Stress distributions in the adhesive layer of a cylindrical assembly AU 2024 T3 - AV 119 - VE ±45°: b) The shear stress (τ_{rz}).

4 Conclusions

The objective of our study was to entirely develop analytical models for dimensioning adhesive-bonded joints. To this end we placed ourselves in the case of a cylindrical assembly.

The basis of our analytical model was the analysis of the stresses applied to an elementary volume of the assembly under consideration, observing the boundary conditions, the geometry and materials of the assembly. The application of an energy method made it possible to obtain the solution of the problem in stress in any point of the structure. The behaviour law enabled us to obtain the deformations then, by integration, the displacements.

The problem in stress, deformation and displacements was thus entirely defined.

The model validation is presented by comparison with finite elements models. For the assembly, the total force-displacement behaviour is well defined. Thus the analytical model makes it possible to determine the rigidity of the assembly and to obtain a simple formulation very quickly, which gives the total behaviour of the assembly. The analytical model underestimated the stresses in the adhesive leading to an over-estimate of the rupture forces. However, this model is reliable and allows fast analysis of this type of assembly.

References

[1] Volkersen, O. „Die Nietkraftverteilung in Zugbeanspruchten Nietverbindungen mit Konstanten Laschenquerschnitten“. *Luftfahrtforschung*, Vol. 15, pp. 41-47, 1938.

[2] Lubkin, L., Reissner, E. “Stress distribution and design data for adhesive lap joints between circular tubes”. *Trans. of ASME, Journal of Applied Mechanics*, Vol. 78, pp. 1213-1221, 1956.

[3] Alwar, R.S., Nagaraja, Y.R. “Viscoelastic analysis of an adhesive tubular joint”. *Journal of Adhesion*, Vol. 8, pp. 76-92, 1976.

[4] Shi, Y.P., Cheng, S. “Analysis of adhesive-bonded cylindrical lap joints subjected to axial load”. *Journal of Engineering Mechanics*, Vol. 119, pp. 584-602, 1993.

[5] Nemes, O., Lachaud, F., and Mojtabi, A. “Contribution to the study of cylindrical adhesive joining”. *International Journal of Adhesion & Adhesives*, Vol. 26, pp 474-480, 2006.

[6] Nemes, O. “Contribution a l’étude des assemblages colles cylindriques et plans“. *Ph.D. thesis*, INSA, Toulouse, 2004.