A HYBRID ELASTIC WAVE METHOD FOR MEASURING ELASTIC CONSTANTS OF FIBER REINFORCED COMPOSITE PLATE

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SUMMARY: In this paper, we developed a hybrid elastic wave method to determine the anisotropic constants of a thin fiber-reinforced composite plate. The method is based on the measurements of the pure mode bulk ultrasonic wave velocities as well as the Lamb wave velocities. The formulae of the pure mode plane wave propagation in an orthotropic medium are summarized, first. Then, the numerical results of Lamb wave dispersion in a thin fiber reinforced plate are presented. In the experimental part, the conventional ultrasonic velocity measurements were conducted to measure the pure mode velocities of a unidirectional fiber reinforced composite and obtain the three corresponding elastic constants. The remaining elastic constants of the unidirectional composite plate were then obtained by using the inverse analyses of the Lamb wave dispersion.

KEYWORDS: Fiber reinforced composite, laser ultrasonics, Lamb wave, anisotropy

INTRODUCTION

Fiber reinforced composite materials have been utilized extensively in industrial applications, such as ship and aircraft engineering. Recently, it has also been applied to the retrofit of concrete structures. The key feature of fiber reinforced composite material is its high strength to weight ratio. The anisotropy of the material allows the strength to be utilized in a given design direction without the addition of extra weight resulting from strength in unnecessary directions. However, the complex constitutive nature of composite materials leads to difficulties in the determination of their mechanical property [1]. By conventional mechanical testing techniques, the Young's moduli along different principal axes of symmetry of fiber-reinforced composites must be measured through different uniaxial tests. In addition to the Young's moduli, the measurements of the shear moduli of fiber-reinforced composites are also not easy, the rail shear or torsion tube test must be invoked.

Ultrasonic techniques have been applied successfully in the field of nondestructive evaluation of materials. Since fiber reinforced composite materials are anisotropic in nature, the conventional ultrasonic technique for measuring material constants in isotropic materials can not be utilized to obtain all of the anisotropic elastic constants. In the literature, there are extensive researches related to the study of ultrasonic waves in composite laminates [2-8]. Although there are some researches studied the measurements of elastic properties of thin plates using Lamb waves [9-12], most of the test specimens are isotropic.

In this paper, we developed a hybrid elastic wave method to determine the anisotropic constants of a thin fiber-reinforced composite plate. The method is based on the measurements of the pure mode bulk ultrasonic wave velocities as well as the Lamb wave velocities. In the following, the formulae of the pure mode pane wave propagation in an orthotropic medium are summarized, first. Then, the numerical result of Lamb wave dispersion in a thin fiber reinforced plate is presented. In the experimental part, the conventional ultrasonic velocity measurements were conducted to obtain the pure mode velocities of a unidirectional fiber reinforced composite and then the corresponding elastic constants. Finally, the laser ultrasonic experiments were conducted to obtain the Lamb wave dispersion and an inverse algorithm was utilized to determine the remaining two elastic constants of the thin unidirectional composite plate.

BULK ULTRASONIC WAVE MEASUREMENT

Pure mode waves in a unidirectional thin fiber reinforced composite laminate

A unidirectional fiber reinforced composite thin plate can be assumed as in transversely isotropic symmetry, and hence, the independent elastic constants are reduced from nine (orthotropic) to five. Let the x_1-axis be pointed along the fiber direction, then the $x_2 - x_3$ plane is the isotropic plane (Figure 1). The nonvanishing elastic constants are

$$[C_{ij}] = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{33} & C_{23} & 0 & 0 & 0 \\ C_{12} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{55} \end{bmatrix} \quad (1)$$

where $C_{44} = (C_{33} - C_{23})/2$.

It is understood that for wave propagation in such an anisotropic plate, pure wave mode (longitudinal or transverse) exists only when the propagation direction is along or perpendicular to the fiber direction. For all the other propagation directions, the wave modes are quasi-longitudinal or quasi-transverse. We note that for bulk ultrasonic wave measurement in such a thin plate, the only accessible plane is the $x_1 - x_2$ plane (see Figure 1), and therefore, it is not feasible to measure the phase velocity with propagation direction along $x_1 -$ or $x_2 -$ direction. For wave normal pointed along the $x_3 -$direction, the relations between the phase velocities and the elastic constants can be obtained as

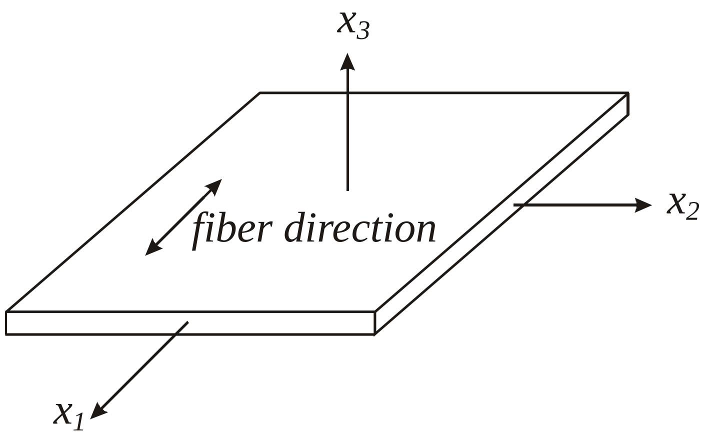

Fig. 1 Coordinates of the composite plate

$$V_{31} = \sqrt{\frac{C_{55}}{\rho}} \ , \quad V_{32} = \sqrt{\frac{C_{44}}{\rho}} \ , \quad V_{33} = \sqrt{\frac{C_{33}}{\rho}} \qquad (2)$$

In the above equation, V_{31} and V_{32} are transverse (shear) wave speeds with polarization directions along the x_1- and x_2-directions, respectively. V_{33} is the longitudinal wave speed.

From Equation (2), we note that three out of five independent elastic constants of a transversely isotropic material can be determined from the conventional ultrasonic bulk wave measurements. We note that with the current available ultrasonic velocity measurement technique, the aforementioned three wave speeds can be measured accurately. The purpose of this paper to utilized the Lamb wave propagation in a thin plate to determine inversely the remaining unknown elastic constants C_{11} and C_{12}.

DISPERSION OF LAMB WAVE IN A THIN TRANSVERSELY ISOTROPIC PLATE

Different from an infinite space, thin plate preserves two parallel flat boundaries, the elastic wave motions on each surface will interact to produce Lamb waves. The phase velocity of Lamb wave is varied according to the distance between the two boundaries, and therefore, the wave is dispersive. The basic modes of Lamb waves are the symmetric and the antisymmetric modes. It is worth noting that for receiving distance sufficiently far away from the source, most of the Lamb wave energy is dominated by the first antisymmetric mode [13]. This means that the dispersion of the first antisymmetric Lamb mode can be obtained directly from the spectral analysis of time domain Lamb wave signal in a thin plate with reasonable accuracy.

In this section, we calculate the phase velocity dispersion of Lamb wave propagation in a thin unidirectional fiber reinforced composite. Two cases are considered, one is for Lamb wave propagating along the fiber direction, the other one is for Lamb wave propagating normal to the fiber direction. A general-purpose computer program for the calculation of phase velocity dispersion in a multi-layered anisotropic medium was utilized. The program was based on the sextic formalism of Stroh [14,15] for the calculations of dispersion curves of anisotropic multi-layered media.

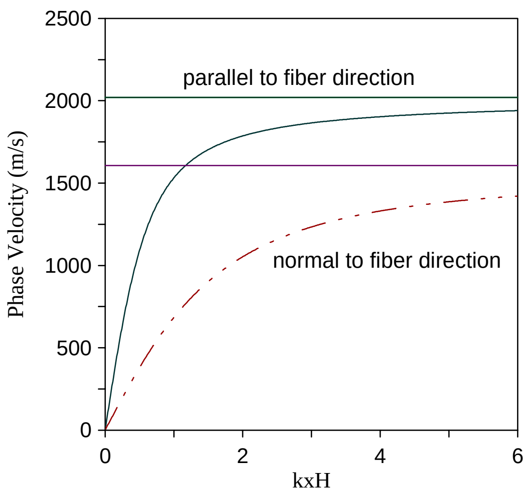

Fig. 2 Dispersion of the Lamb wave

Shown in Figure 2 are the dispersion curves for the anti-symmetric Lamb waves propagating parallel (solid line) and normal (dotted line) to the fiber direction. The material properties of the unidirectional carbon fiber reinforced composite utilized were adopted from reference [7]. The thickness of the plate is H. The results show that for both Lamb wave propagation directions (parallel or normal to the fiber directions), the high frequency component propagates faster than the low frequency component, which is similar to that of the isotropic case. In addition, the propagation speed for Lamb wave propagating parallel to the fiber direction is faster than that normal to the fiber direction. It is worth noting that as the nondimensional wave number (k_xH) is large, the phase velocity of the Lamb wave approaches to the shear wave velocity with the polarization direction along the Lamb wave propagation direction. For example, the Lamb wave velocity for propagation direction parallel to the fiber direction (x_1) approaches to the shear wave velocity V_{31}.

CONVENTIONAL BULK ULTRASONIC MEASUREMENTS

The carbon fiber reinforced composite specimen used is a 14-prepreg layered-plate with fibers oriented along the x_1-direction. The thickness of the composite plate is 2.25 mm, and the mass density is $1520.4\,\mathrm{kg/m^3}$. As described in the previous section, for a transversely isotropic material, three elastic constants (C_{55}, C_{44}, C_{33}) can be measured easily using the conventional ultrasonic measuring technique. Therefore, we measured the phase velocities for wave normal along the x_3-direction, i.e., V_{31}, V_{32}, V_{33}. Then, from Equation (2) and the known density of the specimen, C_{55}, C_{44}, C_{33} are obtained. In the measurements, longitudinal wave and transverse wave transducers with 5 MHz center frequency were used. The diameter of the transducers is 0.25 in. The measured results are

$$V_{31} = 1810.9\,\mathrm{m/s}, \quad V_{32} = 1431.0\,\mathrm{m/s}, \quad V_{33} = 2735.6\,\mathrm{m/s}$$

The corresponding elastic constants can thus be obtained as

$$C_{22} = C_{33} = 11.38\,\mathrm{GPa}, \quad C_{55} = C_{66} = 4.99\mathrm{GPa}$$

$$C_{44} = 3.11\,\mathrm{GPa}, \quad C_{23} = 5.15\,\mathrm{GPa}$$

LASER ULTRASONIC MEASUREMENTS

An Nd:YAG pulsed laser (Quanta-Ray, GCR-130) (wavelength 532 nm) was utilized to generate elastic wave in the layered specimen. The duration of the laser pulse utilized was 10 nsec and the energy carried was about 100 mJ. The composite specimen was rest on a precision translation stage to accurately control the distance between the source and the receiver. An NBS conical transducer was utilized to measure the generated elastic wave signals from the laser sources. The received voltage signals from the conical transducer were then amplified by a preamplifier and recorded by a digital oscilloscope. A trigger signal synchronized with the laser source was utilized to trigger the digital oscilloscope. The recorded signals were sent to a personal computer via GPIB.

Lamb wave in a uni-directional composite thin plate

Figure 3 show the waveforms of Lamb waves propagating parallel to the fiber direction with the source-receiver distance varied from 3cm to 7 cm. These waveforms showed that the

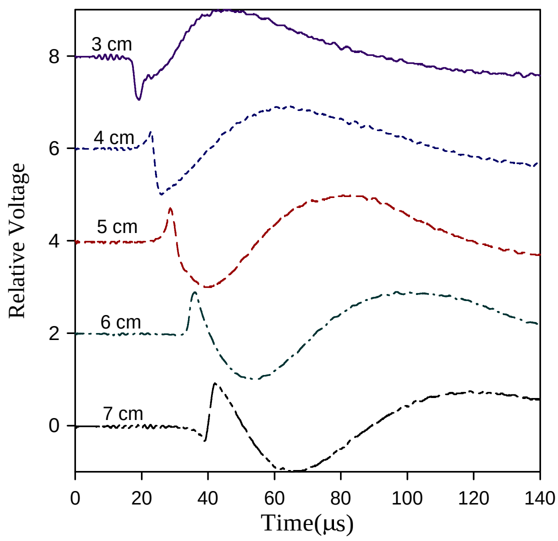

Fig. 3 Lamb wave parallel to the fiber direction

normal displacement of the Lamb wave is dominated by the first order antisymmetric mode. The phase difference ϕ of two wave signals received at two different positions x_1 and x_2 is equal to the phase angle of the complex function S_{x_1,x_2} defined as

$$S_{x_1,x_2}(f) = R_1(f)R_2^*(f) \tag{3}$$

In the above equation, $R_1(f), R_2(f)$ are the frequency spectrums of the wave signals received at two different positions x_1 and x_2. f is the frequency of the harmonic wave and R_2^* denotes the complex conjugate of R_2. The phase velocity v can then be obtained from the relation

$$v = 2\pi\, f\,\frac{x_1 - x_2}{\phi} \tag{4}$$

Utilization of Eqs. (3) and (4), the dependence of the phase velocity of the Lamb wave on the frequency can be obtained and are shown as the square symbols in Figure 4. The solid circles in Figure 4 are the dispersion of Lamb wave that is propagating perpendicular to the fiber direction. We note that the experimental results do reflect the dispersion relations of the theoretical results shown in Figure 2.

INVERSION OF ANISOTROPIC ELASTIC CONSTANTS

For the coordinate system shown in Figure 1, we have shown that C_{33}, C_{44} and C_{55} of a transversely isotropic material could be measured using the conventional bulk ultrasonics. The remaining unknown elastic constants are C_{11} and C_{12}. For wave propagates in the x_2-direction, it is easy to show that C_{11} and C_{12} do not appear in the Christoffel equation and hence the dispersion relation either. Therefore, only the experimental dispersion relation for Lamb wave propagating along the fiber direction (the upper curve with square symbols in Figure 4) can be utilized to determine C_{11} and C_{12}. The lower curve with solid circles in Figure 4 is the Lamb wave dispersion perpendicular to the fiber direction.

The simplex method [16,17] was utilized to determine inversely the elastic constants

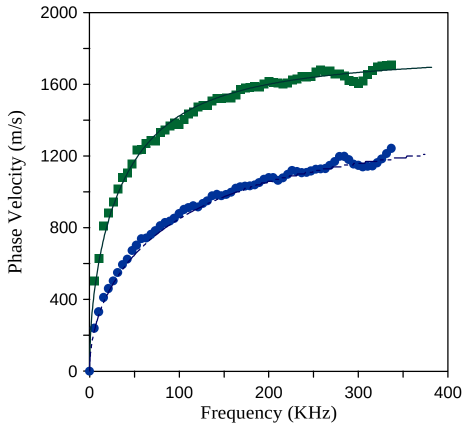

Fig. 4 Lamb wave dispersion in a thin UD-FRP

C_{11} and C_{12}. Since there are two unknowns to be determined, three initial guesses have to be made. To test the influence of the initial guesses on the inversely determined results, four different sets of initial guesses were used. Although the initial guesses of C_{11} and C_{12} are ranging from around 60% to 200% of the true values, the inverse results do converge nicely. The average values of the inversely determined constants C_{11} and C_{12} are $C_{11} = 134.45\,\text{GPa}$, $C_{12} = 7.96\text{GPa}$. The standard deviation of C_{11} is 0.17% of the average value, while that of C_{12} is 2.4%. Therefore, on combining with those measured by the ultrasonic bulk wave method, the elastic constants of the uni-directional fiber reinforced composite laminates are determined as

$$[C_{ij}] = \begin{bmatrix} 134.45 & 7.96 & 7.96 & 0 & 0 & 0 \\ 7.96 & 11.38 & 5.15 & 0 & 0 & 0 \\ 7.96 & 5.15 & 11.38 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.11 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4.99 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4.99 \end{bmatrix} \text{Gpa} \qquad (11)$$

CONCLUSION

Based on the results of this study, we demonstrated the anisotropic elastic constants of a unidirectional fiber reinforced composite plate could be determined using a combination of the ultrasonic bulk wave and Lamb wave. Three out of five elastic constants of the specimen were obtained using the measured pure mode bulk ultrasonic wave velocities. Dispersion of Lamb wave in the thin anisotropic specimen was measured and combined with an inverse algorithm to obtain the rest of the elastic constants. The results showed that the inversely determined Lamb wave dispersion is in good agreement with the experimental one, and this demonstrates the correctness of the inversely determined elastic constants. It is worth noting that in the unidirectional composite, only dispersion for Lamb wave propagating along the fiber direction is intimately related to the elastic constants C_{11} and C_{12}. Finally, we note that with this hybrid method, some of the anisotropic constants can be measured accurately with the well developed bulk wave ultrasonics, while the rest of the undetermined elastic constants can be obtained from the inversion of the Lamb wave dispersion.

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