ADAPTED PROGRESSIVE FAILURE CRITERION FOR MULTI-DIRECTIONAL GRP-LAMINATES

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SUMMARY: The ability to accurately predict the stiffness and strength of laminates composed of glass-fibre reinforced polyester (GRP) is necessary for a sound design when laminates are used in structural applications.
An easily accessible computational tool has been described that is able to go beyond the level of first-ply failure, dealing with the phenomenon of progressive failure.
Our approach is to use the Tsai-Wu criterion to find the critical stress combination in a multi-layer laminate. This failure criterion will be applied to the actual material constitution, taking into account the degraded material properties as for stiffness and strength. The computed stresses are used to determine the mode of failure. The selective criterion that we will use resembles Puck's criterion, extended with a selective parameter.
The computational results have been compared with biaxial tests on flat cross-shaped specimens provided with a small circular hole in the centre. This comparison shows an excellent agreement between computational and experimental results.

KEYWORDS: glass-fibre reinforced polyester, biaxial testing, progressive failure

INTRODUCTION

Structural applications of fibre reinforced composites in civil engineering are still rather limited, due to the high material cost and the hesitation of manufacturers to invest in advanced equipment. Another reason for lagging behind other disciplines (like aeronautical engineering and mechanical engineering) is the lack of familiarity of many building engineers with the potential of fibre reinforced composites for structural applications. The design of structures composed of fibre reinforced composites in civil engineering applications is often based on the assumption that the material is isotropic, ignoring the structural advantages of anisotropic composites.
Although present design codes often refer to the Tsai-Wu criterion, the additional advantages of anisotropy are hardly taken into account and no design tools are available to optimise the composition of multidirectional laminates. A failure approach where the laminate is considered failed when the first layer has failed, will not lead to the application of a sound safety-factor. This will inevitably lead to improper laminate design and uneconomical use of material.

In order to contribute to filling the above-mentioned gap we have developed a computational tool, supported by experiments, that enables to analyse glass-fibre reinforced polyester (GRP) laminates beyond the level of first-ply failure up to ultimate laminate failure.
Biaxial tests have been performed on flat cross-shaped specimen provided with a small circular hole in the centre. The choice to perform tests on flat plate specimens was made mainly in order to achieve in-plane stresses without out-of-plane complications.
The application of a centre hole turned out to be a necessary tool to achieve failure initiation at the inside part of the specimen instead of at the outside edges.
Specimens with different laminate lay-up have been tested, of which the most important one is laminate type A: $(0°, +45°, -45°, 0°)_S$. The external loading has been applied in a variation of biaxial ratios.
It was already known from earlier biaxial experiments, that ultimate failure was approximately three times as high as the level where, according to the Tsai-Wu failure criterion, first-ply failure was predicted.
The experimental results illustrate the importance of dealing extensively with the phenomenon of progressive failure and the need of an easy accessible computational tool to predict deformation and ultimate failure of laminates in structural applications.
The parameters used in the computational procedures, using the Ansys finite element package, have been adapted to a value for which a satisfactory result is achieved for different types of testing.

FINITE ELEMENT ANALYSIS

Laminate Failure Model

The mechanisms for damage progression and accumulation in laminated composites containing stress concentrations are extremely complicated. They are a combination of matrix cracking, fibre/matrix splitting, fibre breakage and delamination, etc. It is also critical in the design of a composite to be able to select criteria that can appropriately handle damage phenomena and to predict residual strengths of the laminates after they have been preloaded beyond the initial failure level.
Our approach is to use the Tsai-Wu criterion to find the critical stress combination in a multi-layer laminate. This failure criterion will be applied to the actual material constitution, taking into account the degraded material properties like stiffness and strength.
The computed stresses are used to determine the mode of failure. The selective criterion that we will use resembles Puck’s criterion, except that only one particular type of failure can occur simultaneously at a specific load level. For this purpose, we use the multiplying factor b as a selective parameter. A proper value for parameter b will be determined in accordance with the experimental results.
To determine the type of failure, the stress level relative to the ultimate stress in the fibre direction and perpendicular to the fibres, defined through Π_1 and Π_2 in Eqn (1) and Eqn (2), has to be compared.
The factors Π_1 and Π_2 are different for tensile and compressive stresses:

$$\Pi_1 = \frac{s_1}{X_t} \quad (s_1 \geq 0) \qquad \Pi_1 = \frac{s_1}{X_c} \quad (s_1 < 0) \tag{1}$$

$$\Pi_2 = \sqrt{\left(\frac{s_2}{Y_t}\right)^2 + \left(\frac{s_6}{S}\right)^2} \quad (s_2 \geq 0) \qquad \Pi_2 = \sqrt{\left(\frac{s_2}{Y_c}\right)^2 + \left(\frac{s_6}{S}\right)^2} \quad (s_2 < 0) \tag{2}$$

If $\Pi_1 > b.\Pi_2$ we will assume fibre failure occurs. In the opposite case ($\Pi_1 \leq b.\Pi_2$), we will assume matrix failure occurs.

Progressive Failure Analysis

The procedure to analyse progressive failure consists of a number of sub-procedures that has to be followed:

i. Generation of a finite element model
ii. First solving step and determination of first ply failure
iii. Modification of elements and repeated solving steps
iv. Output of results

The first important step is the generation of the finite element model in Ansys 5.3, the choice of type of element (Shell91), the element key options (element geometry, number of layers, output options, etc.) and the definition of material properties.
The generation of the finite element model can be realised with a user-friendly graphic interface. The mesh generation can be generally specified and refined locally at a user-specified manner.
Most important for the proposed procedure of analysing progressive failure are the properties of the applied materials.
Because we will apply classical lamination theory, we can restrict the material properties to the in-plane stiffness, Poisson's ratios and strengths.
For each stage of damage, a separate material number is defined.
Material-1 is undamaged material.
Material-2 is damaged by matrix failure along the direction of the fibres. Compared with the undamaged material, the properties in the fibre direction are not changed, but perpendicular to the fibres the stiffness has been reduced to zero.
Material-3 is defined as completely failed: the stiffness equals zero and the strength values are set values that are one order of magnitude greater. This means that when using the Tsai-Wu failure criterion on the failed layer, failure will not be found again for the same layer.

First, a unit load is applied to the finite element model. After the problem has been solved, the inverse safety factor $x_3 = 1/Sf$ is determined for each element, according to the Tsai-Wu failure criterion. The inverse of the determined maximum for x_3 equals the safety factor of the laminate. The unit load multiplied with the safety factor Sf, is the load that has to be applied to the structure in order to find a critical value for the Tsai-Wu criterion. Therefore, at this level of loading first-ply failure will computationally occur.
From this point on, a reiterative procedure will be followed to accomplish a gradually degrading of the model until ultimate failure will have been reached.
The stresses in the integration points of the critical element layer are used as input for the Tsai-Wu criterion. The stresses in the normative integration point are used as input for the modification procedure.
Dependent on the actual material number of the critical layer and the result of the comparative criterion based on Eqn (1) and Eqn (2), the material will be modified.
The adjusted finite element model will be solved first in order to determine the external loading that has to be applied to reach the next occurrence of a critical layer. At the same time, the condition of external loading equal to or greater than external loading in the previous solving step must be fulfilled.
Solving the model with this external loading followed by material modification concludes the iterative solving procedure.

BIAXIAL EXPERIMENTS

Biaxial Test Set-up

Because of the experimental potential of the laboratory, we have chosen a test set-up based on the flat plate method, where loading can be applied in two mutually perpendicular directions. A variety of biaxial states of stress (in the tension-tension stress space) can be achieved by rotating the principal material axes with respect to the loading directions.
A circular test area with a 100mm diameter has been chosen, to ensure stress homogeneity within a reasonable test section. Towards the outside of the specimen, extra laminate thickness was applied, enabling the introduction of loads via tapered tabs.
Despite this specimen configuration, failure was still initiated at the outside corners. Applying edge reinforcements, such as attached aluminium plates, could not solve this problem.
It could be concluded from a great number of tests, that there is no other alternative to investigating this type of flat biaxial specimen, than to apply a hole in the centre.
The radius of the hole was chosen small enough so as not to disturb too much the stress-homogeneity of the centre area. Moreover, we achieved with the small radius that the stresses along the edge of the hole are of a higher level than at the outside edges of the corner regions.
A specific advantage of testing notched biaxial specimen is that a particular laminate lay-up under investigation is loaded with a great variety of states of stress.
The most critical combination of the state of stress and the orientation of the laminate with respect to the edge of the hole will result in initiation of failure. There seems to be no other way to investigate laminates under biaxial stress with in-plane loading on a flat specimen, with failure initiation having to start from the inner part of the specimen.

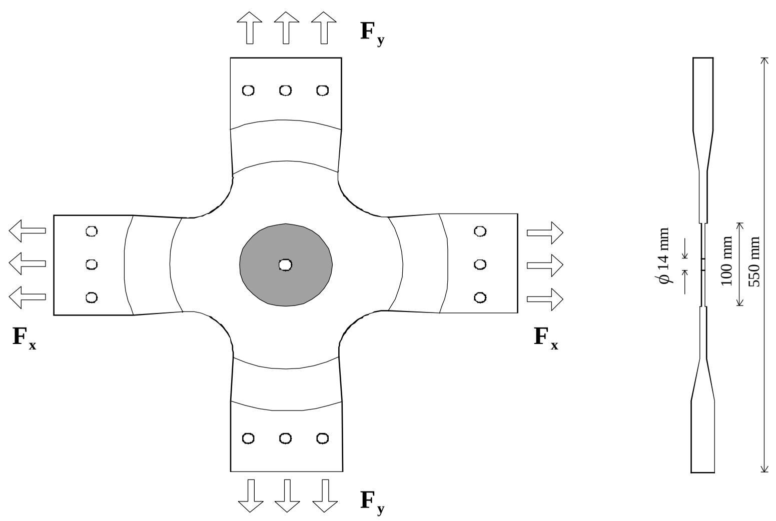


Fig. 1: Biaxial loaded specimen.

The test set-up consists of two perpendicular intersecting loading-frames. In each frame, a hydraulic jack can independently apply the loading.
Normally there are four hydraulic cylinders needed in a biaxial test configuration, to make sure that there is no resultant displacement of the centre of a flat plate specimen during loading. The test set-up in our case was designed in such a way, that only two hydraulic closed loop systems were needed. Both systems consist of a hydraulic cylinder and a load cell built into mutually perpendicular loading frames and controlled with one function generator allowing for a constant loading ratio.

The two frames intersect. One of the frames is suspended from a four-meter high portal, allowing horizontal displacements. These horizontal displacements will cause reaction forces at the clamped specimen due to gravity. However, because of the height of the suspension portal and the limited displacement, the reaction forces are negligible.
The signal of the load cells is generated by built-in strain gauges. These strain gauge readings are subsequently used for recording the exact loads applied to the specimen and as feedback signals for controlling the pressures by means of the servo-hydraulic system used.
In order to avoid out-of-plane bending of the specimen during loading, the laminate should be symmetric with respect to the stacking sequence. To avoid in-plane rotation, the direction of loading was restricted to the symmetry directions of the laminate.
The biaxial tests have been performed in a load-controlled manner, while the ratio of forces in both the x- and y-direction was kept constant.

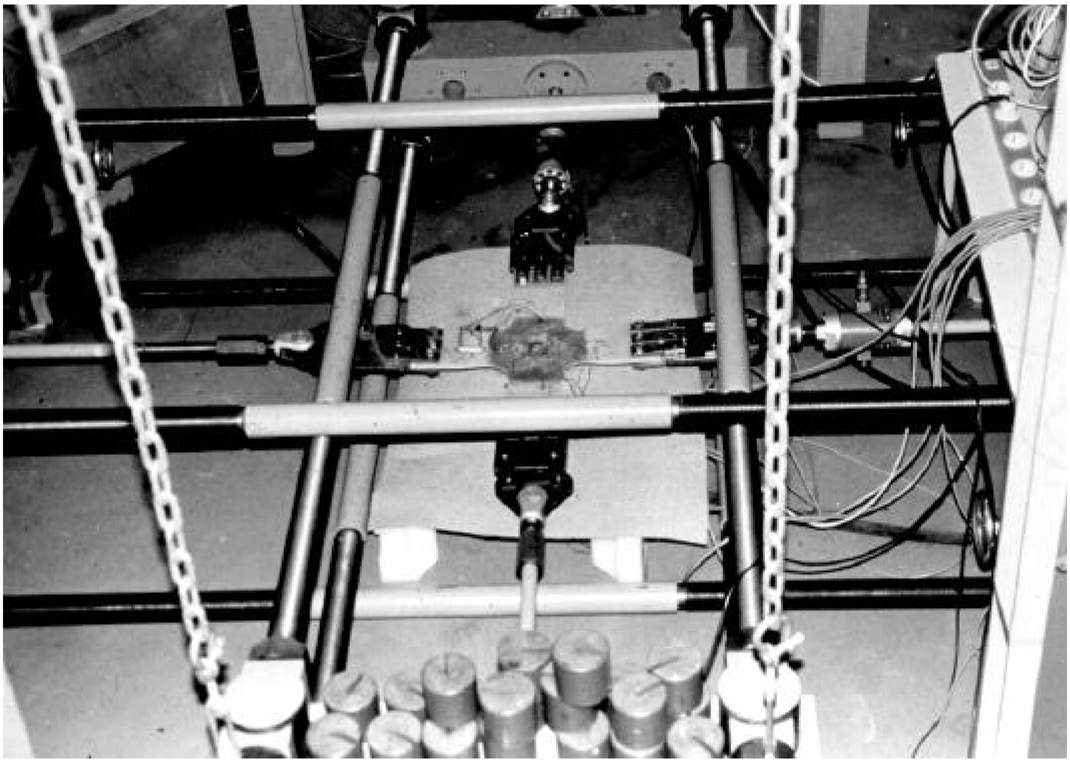


Fig. 2: Biaxial loading frames.

COMPUTATIONAL VERSUS EXPERIMENTAL RESULTS

The final goal of our approach is to be able to reliably predict the behaviour of multi-layer composites under various loading conditions. To that end, we have developed a computational tool, enabling the establishment of a sound and optimal design for composite laminates. The central issue of the computational tool is the monitoring of successive failure of separate layers according to the Tsai-Wu criterion.
The proposed procedure has been simulated with the Ansys finite element package. The computational method as developed, implicitly needs a number of parameters that have to be set to proper values in order to follow the results of the biaxial experiments as closely as possible.
The dimensioning of the biaxial specimen was realised in such a way that ultimate failure initiated at the inside hole of the central circular area of the specimen. Within a test series with

the same type of laminate and loading ratio in the x- and y-direction, little scatter was found in the ultimate loading data where final rupture was reached. So the conclusion is justified that the test set-up, as well as the specimen configuration, are representative.
Moreover, we observed no delamination phenomena along the edge of the inside hole despite the fact that failure initiated at this location.
From these facts, it may be concluded that there is a minimal effect of normal out-of-plane stresses at the edge of the hole. Therefore, we think the application of classical lamination theory is justified for our specimen dimensions, choice of material and test set-up.

The main objective is to find out, whether the described computational method is capable of simulating the mechanical behaviour of multi-layer composites in laboratory testing and of finding the most appropriate parameters.
In order to achieve this goal, the proposed computational procedure have been performed for a number of laminate types and different loading ratios in the two loading directions, coinciding with the variations applied during biaxial testing.
The results will be used as a reference for the input parameters involved in the prediction of different stages of damage, such as first-ply failure, first fibre failure and ultimate laminate failure.
The analyses are performed on a precise model of the biaxial specimen. The mesh refinement that we used is high enough to get reliable results within an acceptable computation time.
The distributions and the increase of the Tsai-Wu factor x_3 during the stage of first-ply failure are depicted in Figure 3a to Figure 3d, showing the phenomenon of progressive failure close to the edge of the centre hole.

The computational results for the successive loading steps have been evaluated by counting the elements where matrix failure or fibre failure did occur. From these numbers, the development of damage can be resolved.
One of the major output parameters that we use, is the total number of layers in which fibre failure does occur. An extra-ordinary increase of the total number of layers with fibre failure will be considered as ultimate laminate failure (ULF). Parameter Δff is defined as the value taken for this extra-ordinary increase.

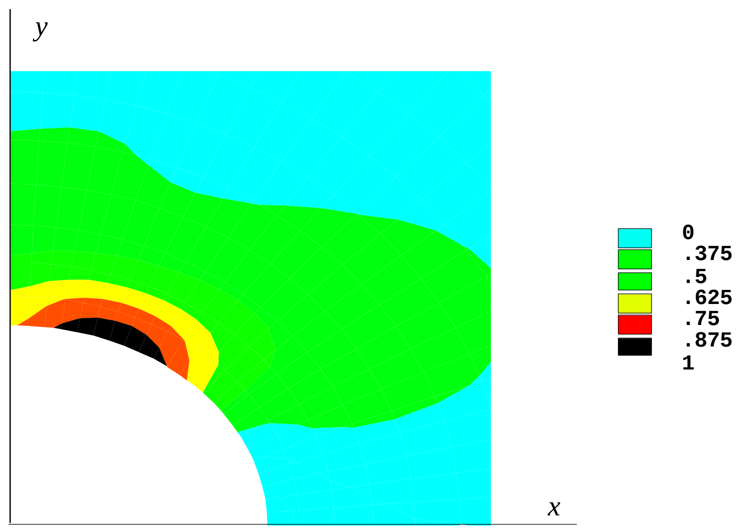


Fig. 3a: Distribution of Tsai-Wu factor x_3, prior to first-ply failure.

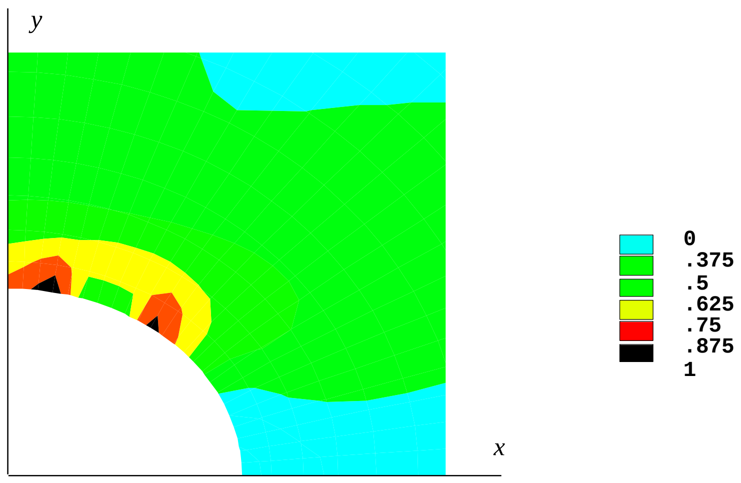


Fig. 3b: Distribution of x_3 after first-ply failure.

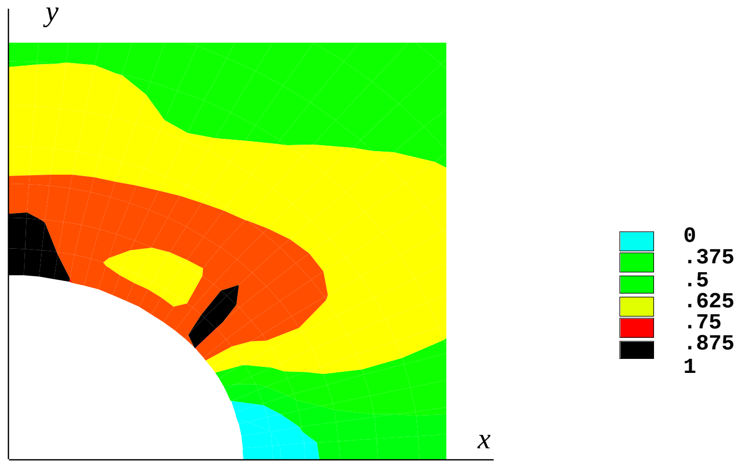


Fig. 3c: Distribution of x_3 (continued).

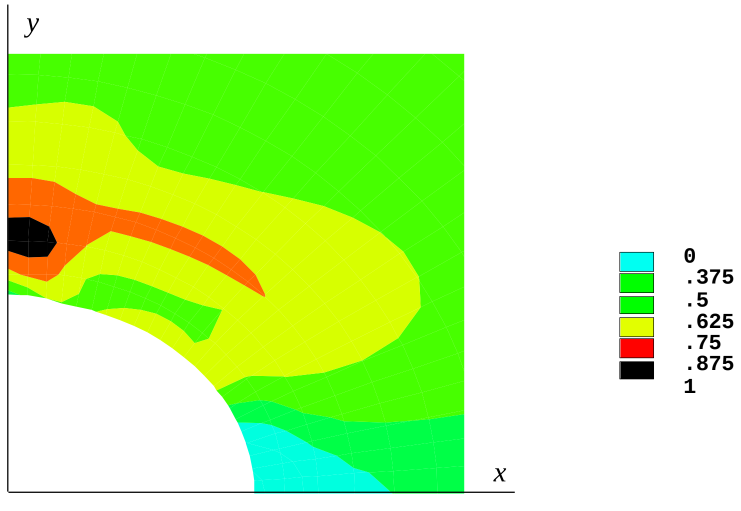
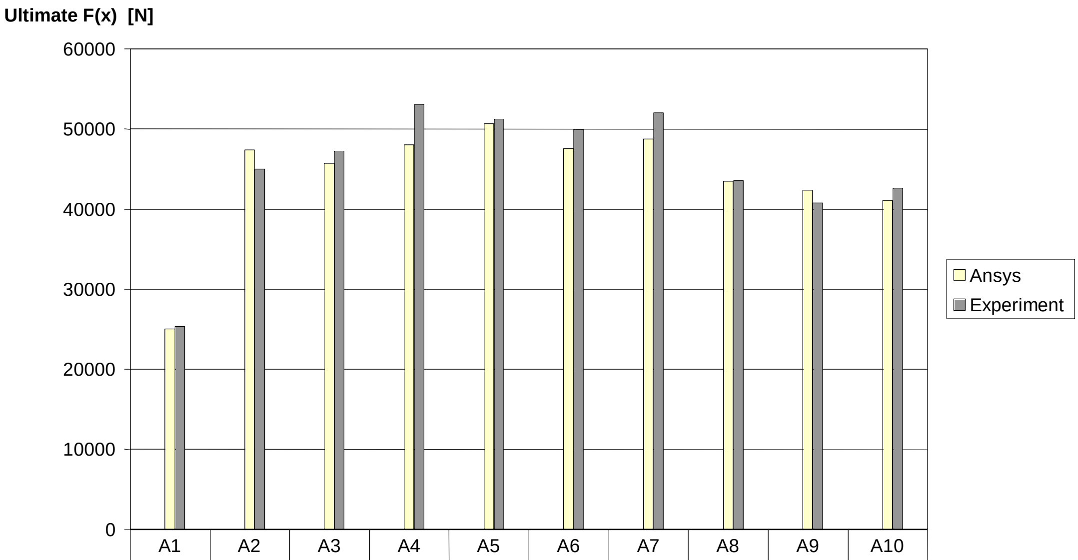


Fig. 3d: Distribution of x_3 (continued).

There is of course a direct relation between the area covered by elements where fibre failure will occur and the mesh refinement (defined by parameter mf). With a higher mesh refinement, more elements will fail at the same time within a comparable area. This will affect

both the number of elements where first fibre failure did occur and the increase of the number of elements that failed through fibre failure.
We use the initial uniaxial strengths and the average stiffness to compute all types of biaxial testing with different values for the parameters mf , b , $F_{12}(1)$ and $F_{12}(2)$.
As an example, the comparison between computational and experimental results for laminate type A are depicted in Fig. 4 for a specific combination of parameters.
We have combined highly similar tests (F_y/F_x almost the same) because full computational analyses of the local stresses in all the specimen would be too time consuming, and would yield little extra information.
Parameter a $(0 < a < 1)$ equals the F_y/F_x -ratio. So, a measures the extent of the bi-axiality of the loading.
The results show that our computational tool is able to follow the deformation and progressive failure behaviour of composite laminates.



	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
α	0.976	0.492	0.453	0.410	0.374	0.334	0.254	0.103	0.078	0.055
Ansys	25020	47446	45709	48047	50710	47588	48788	43509	42328	41126
Experiment	25420	45030	47260	53080	51200	49923	52000	43560	40750	42620

Fig. 4: Bar graph comparing computational and experimental results for parameter set: $b = 20$, $F_{12}(1) = 0$, $F_{12}(2) = 0$, $mf = 3$, $\Delta ff = 3$. $(a = F_y/F_x)$

In order to illustrate the agreement between computational and experimental results for a particular test, both results are compared with respect to the evolution of the tensile force F_x as a function of the strain in the x-direction.
For this comparison, we adopt the average strain computed at the integration points coinciding with the area of the attached strain gauges as the computational strain.
The agreement between the experimental results and the corresponding computational analysis is remarkably good, as illustrated in Fig. 5.
The Ansys analysis was performed up to a level where the overall computational damage is very high. At this level, the material in the computational model is still capable to transfer loads carried by the remaining intact part of the structure. The real specimen, however, will fail immediately at this point, because of the limitations of force-control in the closed loop testing circuit.

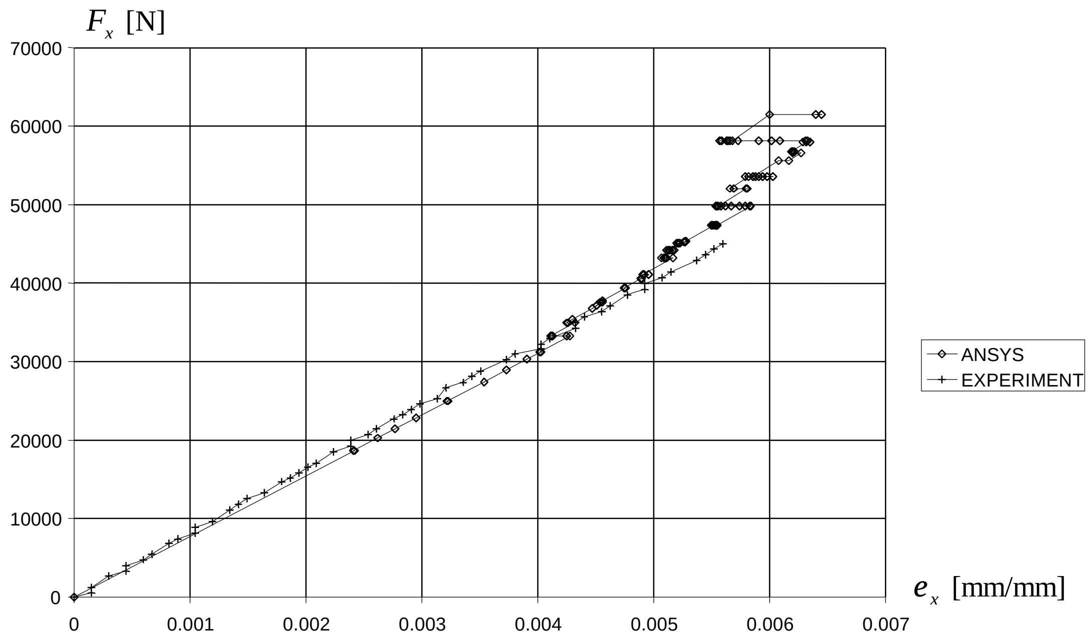


Fig. 5: Comparing Ansys computational and experimental results $(a = 0.49)$.

CONCLUSIONS

We have developed a finite element computational tool for simulating progressive failure and ultimate performance of laminates, based on the deformation and failure mechanisms per unidirectional layer.
The reliability of this computational tool is verified by biaxial experiments on glass-fibre polyester laminates. Despite the use of a rather simple procedure to select the type of failure that will occur at any stage of loading, the computational procedure proved capable of reasonably predicting ultimate failure.
The agreement between experiments and the computational analysis was remarkably good for a final set of parameters.

Successive failure stages were computed for various laminate types and loading ratios using several parameter combinations.
The best agreement between the computational and the experimental results for different loading ratios is achieved for a specific laminate with three fibre directions (type A) and with the following set of parameters: $mf = 3$, $b = 20$, $F_{12}(1) = 0$, $F_{12}(2) = 0$.
The computational agreement for laminates with less than three fibre directions is not perfect. The computational results over-estimate the experimental results. A major reason for this is quite likely the harmful effect of transverse inter-laminar stresses near the edge of the centre hole, where failure is initiated. These stresses have to be quantified yet in order to proof the statement above.
The chosen mesh refinement will certainly affect the overall result of a progressive failure analysis. There will be an optimal mesh refinement for any structure, correlated with the available computation time and the aimed exactness of the solution.
Our aim has been to show that our finite element computational tool, using the Tsai-Wu failure criterion for the unidirectional layer, is able to follow the deformation and progressive failure behaviour of composites. The comparison between experimental and computational results shows that we have been successful in that respect.

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