AEROSPACE COMPOSITE LATTICE STRUCTURES

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SUMMARY: The paper is concerned with design and fabrication of composite lattice structures made from carbon and aramid epoxy composite materials by automated filament winding. An integrated process resulting in high-performance lattice structures and including interrelated design, analysis, fabrication and testing procedures is described in application to cylindrical launch vehicle interstages loaded with axial compression. Governing equations, design conceptions, and fabrication methods are presented for typical lattice structures whose specific features and efficiency are discussed and highlighted with respect to traditional stringer-stiffened and sandwich structures.

KEYWORDS: aerospace applications, composite structures, design, analysis, fabrication.

INTRODUCTION

Composite materials with carbon or aramid fibers and polymeric matrices are known to have high specific strength and stiffness and, in combination with automatic manufacturing processes, make it possible to fabricate composite structures with high level of weight and cost efficiency. As known, the substitution of metal alloys by composite materials, in general, reduces structure mass by 20 –30 %. However, in some cases the combination of high mechanical characteristics of composites with proper structural conceptions and fabrication processes (which allow these characteristics to be realized with great efficiency) gives a qualitative improvement of structure performance. The widely known example of a structure like this is a composite pressure vessel made by filament winding. The similar effect takes place also with respect to lattice composite structures to be considered here.

Lattice structures are usually made in the form of thin-walled cylindrical or conical shells and consist of systems of helical and circumferential ribs (see Fig.1) with one-side skin, two-side skin or without a skin. They are made by continuous filament winding from graphite and aramid epoxy composites.

Lattice composite structures developed in the early eighties combine well advantages of composite materials with structural and manufacturing conceptions allowing materials

strength and stiffness to be used with proper completeness. Until the present, lattice structures do not have any analogues in cost and weight efficiency that could copmpete them in the class of high-loaded structures. Integral lattice structures that are now under production can have diameter up to 4 m and length up to 8 m. The size of a segmented lattice structure is not limited.

This paper presents the Russian experience in lattice technology. Other attempts to develop analogous structures are described elsewhere [1, 2].

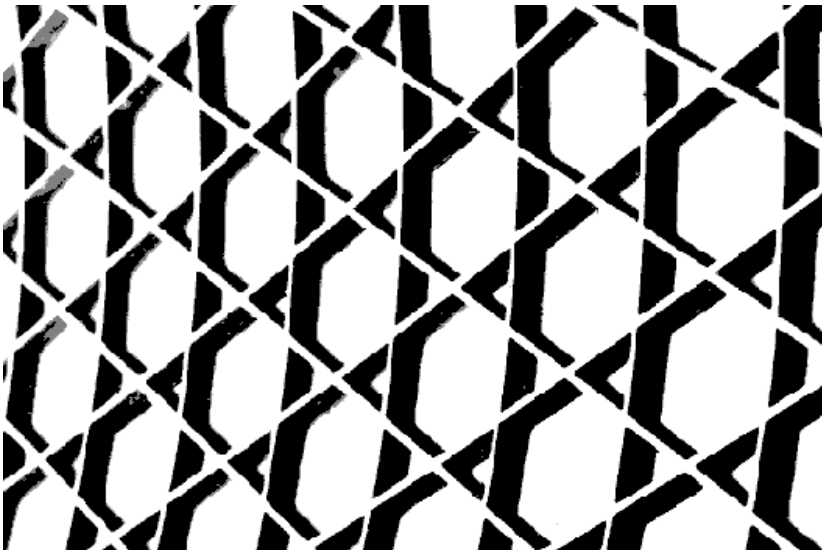

Fig. 1: A typical lattice structure consisting of helical and circumferential ribs

ANALYSIS AND DEVELOPMENT OF LATTICE STRUCTURES

Equations

Consider a cylindrical shell with given radius, R, length, L, and loaded with axial compressive force, P. Geometry of a lattice structure is shown in Fig. 2 where x and y are axial and circumferential coordinates, respectively, while z is the normal coordinate counted from the midsurface of the ribs. In the general case, the structure can have an inner and an outer skins that are considered as membrane layers. The lattice layer shown in Fig. 2 consists of $\pm\varphi$ helical and circumferential ribs the dense and regular system of which is simulated with two-dimensional micropolar continuum [3, 4, 5] alowing us to take into account in-plane bending of ribs.

As a result, constitutive equations can be presented as

$$\left[T_i\right] = \left[S_{ij}\right]\left[e_j\right] \tag{1}$$

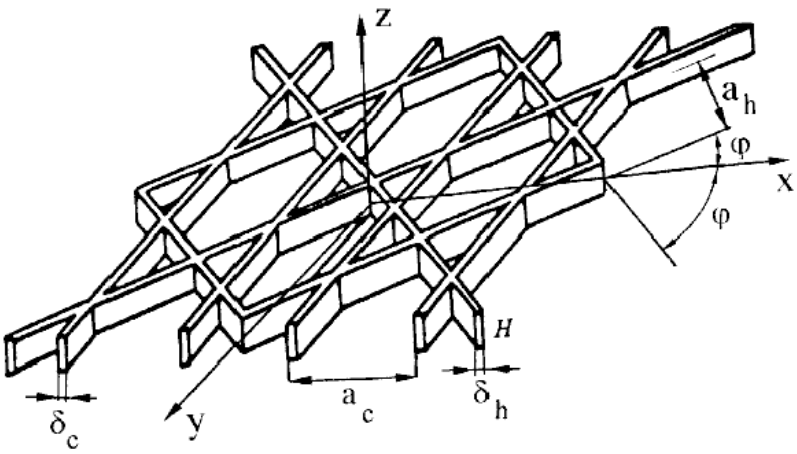

Fig. 2: Structural parameters of the rib system

In Eqn 1, i, j = 1, 2, 3, ...12; $T_1 = N_x$, $T_2 = N_y$, $T_3 = N_{xy}$, $T_4 = N_{yx}$ and $e_1 = \varepsilon_x$, $e_2 = \varepsilon_y$, $e_3 = \varepsilon_{xy}$, $e_4 = \varepsilon_{yx}$ are membrane stress resultants and the corresponding strains; $T_5 = V_x$, $T_6 = V_y$ and $e_5 = s_x$, $e_6 = s_y$ are transverse forces and the corresponding transverse shear deformations; $T_7 = M_x$, $T_8 = M_y$, $T_9 = M_{xy}$ $T_{10} = M_{yx}$ and $e_7 = \kappa_x$, $e_8 = \kappa_y$, $e_9 = \kappa_{xy}$, $e_{10} = \kappa_{yx}$ are stress couples and curvature components; $T_{11} = H_{xz}$, $T_{12} = H_{yz}$ and $e_{11} = \omega_{xz}$, $e_{12} = \omega_{yz}$ are in-plane moments and rotations. Nonzero components of the stiffness matrix in Eqn 1 are

$$S_{11} = a_{11} + b_{12} + B_{11}^{o} + B_{11}^{i} \qquad S_{12} = a_{12} - b_{12} + B_{12}^{o} + B_{12}^{i}$$

$$S_{17} = S_{71} = \frac{H}{2}\left(B_{11}^{o} - B_{11}^{i}\right) \qquad S_{18} = S_{81} = S_{27} = S_{72} = \frac{H}{2}\left(B_{12}^{o} - B_{12}^{i}\right)$$

$$S_{21} = a_{12} - b_{12} + B_{12}^{o} + B_{12}^{i} \qquad S_{22} = a_{22} + b_{12} + B_{22}^{o} + B_{22}^{i}$$

$$S_{28} = S_{82} = \frac{H}{2}\left(B_{22}^{o} - B_{22}^{i}\right) \qquad S_{33} = a_{12} + f_{11} + B_{33}^{o} + B_{33}^{i}$$

$$S_{34} = S_{43} = a_{12} - f_{12} + B_{33}^{o} + B_{33}^{i}$$

$$S_{39} = S_{93} = S_{310} = S_{103} = S_{49} = S_{94} = S_{410} = S_{104} = \frac{H}{2}\left(B_{33}^{o} - B_{33}^{i}\right)$$

$$S_{44} = a_{12} + f_{22} + B_{33}^{o} + B_{33}^{i} \qquad S_{55} = b_{31} \qquad S_{66} = b_{32}$$

$$S_{77} = d_{11} + c_{12} + \frac{H^2}{4}\left(B_{11}^{o} + B_{11}^{i}\right) \qquad S_{78} = S_{87} = d_{12} - c_{12} + \frac{H^2}{4}\left(B_{12}^{o} + B_{12}^{i}\right)$$

$$S_{88} = d_{22} + c_{12} + \frac{H^2}{4}\left(B_{22}^{o} + B_{22}^{i}\right) \qquad S_{99} = d_{12} + c_{11} + \frac{H^2}{4}\left(B_{33}^{o} + B_{33}^{i}\right) \tag{2}$$

$$S_{910} = S_{109} = d_{12} - c_{12} + \frac{H^2}{4}\left(B_{33}^o + B_{33}^i\right)$$

$$S_{1010} = d_{12} + c_{22} + \frac{H^2}{4}\left(B_{33}^o + B_{33}^i\right) \qquad S_{1111} = K_{11} \qquad S_{1212} = K_{22}$$

where

$$a_{11} = \frac{2}{a_h} B_h c^4 \qquad a_{12} = \frac{2}{a_h} B_h s^2 c^2 \qquad a_{22} = \frac{2}{a_h} B_h s^4 + \frac{B_c}{a_c}$$

$$b_{12} = \frac{2}{a_h} s^2 c^2 \left(\frac{1}{C_h} + \frac{l_h^2}{12 I_h}\right)^{-1} \qquad f_{11} = \frac{2c^4}{a_h}\left(\frac{1}{C_h} + \frac{l_h^2}{3 I_h}\right)^{-1}$$

$$f_{12} = \frac{2}{a_h} s^2 c^2 \left(\frac{1}{C_h} + \frac{l_h^2}{3 I_h}\right)^{-1} \qquad f_{22} = \frac{2}{a_h} s^4 \left(\frac{1}{C_h} + \frac{l_h^2}{12 I_h}\right)^{-1} + \frac{1}{a_c}\left(\frac{1}{C_c} + \frac{l_c^2}{3 I_c}\right)^{-1}$$

$$d_{11} = \frac{2}{a_h} D_h c^4 \qquad d_{12} = \frac{2}{a_h} D_h s^2 c^2 \qquad d_{22} = \frac{2}{a_h} D_h s^4 + \frac{D_c}{a_c}$$

$$c_{11} = \frac{2}{a_h} T_h c^4 \qquad c_{12} = \frac{2}{a_h} T_h s^2 c^2 \qquad c_{22} = \frac{2}{a_h} T_h s^4 + \frac{T_c}{a_c}$$

$$b_{13} = \frac{2}{a_h} s^2 \left(\frac{1}{C_h} + \frac{l_h^2}{12 D_h}\right)^{-1} + \frac{1}{a_c}\left(\frac{1}{C_c} + \frac{l_c^2}{12 D_c}\right)^{-1} \qquad b_{31} = \frac{2}{a_h} c^2 \left(\frac{1}{C_h} + \frac{l_h^2}{12 D_h}\right)^{-1}$$

$$K_{11} = \frac{2}{a_h} I_h c^2 \qquad K_{22} = \frac{2}{a_h} I_h s^2 + \frac{I_c}{a_c} \qquad s = \sin\varphi \qquad c = \cos\varphi$$

Here, subscripts “h” and “c” correspond to helical and circumferential ribs,

$B_h = E_h H \delta_h$ and $B_c = E_c H \delta_c$ are axial stiffnesses of ribs,

$C_h = G_h H \delta_h$ and $C_c = G_c H \delta_c$ are shear stiffnesses,

$D_h = \frac{E_h}{12}\delta_h H^3$ and $D_c = \frac{E_c}{12}\delta_c H^3$ are out-of-plane bending stiffnesses,

$I_h = \frac{E_h}{12}\delta_h^3 H$ and $I_c = \frac{E_c}{12}\delta_c^3 H$ are in-plane bending stiffnesses, and

$T_h = \frac{G_h}{12}\delta_h^3 H$ and $T_c = \frac{G_c}{12}\delta_c^3 H$ are torsional stiffnesses of the ribs.

Rib spans l_h, l_c and spacings a_h, a_c are connected by the following relations (see Fig.2):

$$l_h = \frac{a_h}{\sin 2\varphi} \qquad l_c = \frac{a_h}{2\cos\varphi} \qquad a_c = \frac{a_h}{2\cos\varphi} \tag{3}$$

Coefficients B in Eqns 2 are membrane stiffnesses of the skin (superscripts “o” and “i” correspond to the outer and the inner skin, respectively) that can be found elsewhere [6].

Stress resultants and couples acting in the lattice structure should satisfy the following eqilibrium equations:

$$\frac{\partial N_x}{\partial x}+\frac{\partial N_{yx}}{\partial y}+p_x=0 \qquad \frac{\partial N_y}{\partial y}+\frac{\partial N_{xy}}{\partial x}+\frac{V_y}{R}+p_y=0$$

$$\frac{\partial V_x}{\partial x}+\frac{\partial V_y}{\partial y}-\frac{N_y}{R}+p_z=0 \qquad \frac{\partial M_x}{\partial x}+\frac{\partial M_{yx}}{\partial y}+\frac{H_{yz}}{R}-V_x=0$$

$$\frac{\partial M_y}{\partial y}+\frac{\partial M_{xy}}{\partial x}-V_y=0 \qquad \frac{\partial H_{xz}}{\partial x}+\frac{\partial H_{yz}}{\partial y}-\frac{H_{yz}}{R}+N_{xy}-N_{yx}=0$$

where p are surface tractions. Strain-displacement equations are

$$\varepsilon_x=\frac{\partial u}{\partial x} \qquad \varepsilon_y=\frac{\partial v}{\partial y}+\frac{w}{R} \qquad \varepsilon_{xy}=\frac{\partial v}{\partial x}-\psi \qquad \varepsilon_{yx}=\frac{\partial u}{\partial y}+\psi$$

$$\kappa_x=\frac{\partial \theta_x}{\partial x} \qquad \kappa_y=\frac{\partial \theta_y}{\partial y} \qquad \kappa_{xy}=\frac{\partial \theta_y}{\partial x} \qquad \kappa_{yx}=\frac{\partial \theta_x}{\partial y}+\frac{\psi}{R}$$

$$s_x=\frac{\partial w}{\partial x}+\theta_x \qquad s_y=\frac{\partial w}{\partial y}-\frac{v}{R}+\theta_y \qquad \omega_{xy}=\frac{\partial \psi}{\partial x} \qquad \omega_{yx}=\frac{\partial \psi}{\partial y}-\frac{\theta_y}{R}$$

Lattice structures are characterized with six independent kinematic variables — axial and circumferential displacements, u and v, deflection, w, and three rotations, θ_x, θ_y, and ψ, of the structure element around axes x, y and z, respectively.

Design

As was already noted, lattice structures have usually a form of cylindrical or conical shell and consists of systems of helical and circumferential ribs shown in Fig.1. The optimum lattice structure has no skin. However, if the skin is necessary because of structural requirements, the lattice structure can be made with one- or two-sided skin.

The lattice structure (without skins) is specified by the following design variables (see Fig.2):

H – rib height which is the shell wall thickness,

φ – angle between helical ribs and the shell meridian,

δ_h , $\delta_{\tilde{n}}$ – thicknesses of helical and circumferential ribs (it is supposed that the ribs have rectangular cross sections),

a_h , $a_{\tilde{n}}$ – spacings of the ribs counted along normals to their axes for system of helical and circumferential ribs.

Because a_c is expressed in terms of a_h by Eqns 3, there are five independent variables – H, φ, δ_h, δ_c and a_h.

Design of a lattice structure is performed under the condition of minimum mass

$$M = 2\pi RLH\left(2\rho_h \frac{\delta_h}{a_h} + \rho_c \frac{\delta_c}{a_c}\right) \tag{4}$$

where ρ_h and ρ_c are densities of helical and circumferential ribs.

Design constraints are imposed on general buckling of the structure, local buckling of helical ribs and strength of helical ribs in compression. The basic feature of a lattice structure which causes its exceptionally high resistance to axial compression, as the most dangerous kind of loading, is concerned with the structure property to stabilize its shape under load. In consequence, critical load corresponding to general form of buckling, is so high that the loss of structural load-carrying capacity under high levels of compressive loads, as a rule, is concerned with failure of helical ribs under compression rather than with general buckling. As to the local form of buckling for helical ribs (between points of intersections), the corresponding critical load can be controlled by proper arrangement of circumferential ribs. In the basic structure considered here, the intersection points of circumferential and helical ribs devide the spans of helical ribs (rib segments between the points of intersection of symmetric helical ribs) into two equal parts. Circumferential ribs can be arranged in such a way that the length of the ribs parts undergoing local buckling will be
smaller than the corresponding span by 3 or 4 times.

Design of a cylindrical lattice structure loaded with axial compression is performed in two steps. First, the simplified continuum model of the structure ignoring ribs in-plane bending and assuming axisymmetric mode of general buckling is introduced allowing us to arrive at analytical results with the aid of Geometric Programming Method [5, 7]. Results of this first-step design show there exists some critical value of compression strength of helical ribs which can be specified as

$$\bar{\sigma}^* = 028\left(\beta \frac{\rho_h}{\rho_c} \frac{P^2}{R^4} E_h^2 E_c\right). \tag{5}$$

If the actual ultimate strength of material $\bar{\sigma}$ is less than $\bar{\sigma}^*$, the optimal design parameters have the form

$$H = \frac{P}{2R}\ \mathrm{tg}\,\varphi \sqrt{\frac{\beta E_h}{3\bar{\sigma}^3}} \qquad \mathrm{tg}^2\varphi = \frac{6R^2}{PE_h}\sqrt{\frac{\rho_c}{\beta\rho_h}\frac{\bar{\sigma}^5}{E_c}}$$

$$\frac{\delta_h}{a_h} = \frac{1}{\pi \sin 2\varphi}\sqrt{\frac{3\bar{\sigma}}{\beta E_h}} \qquad \frac{\delta_c}{a_c} = \frac{\sin\varphi}{\pi\cos^3\varphi}\frac{\rho_c}{\rho_h}\sqrt{\frac{3\bar{\sigma}}{\beta E_h}} \tag{6}$$

If $\bar{\sigma} > \bar{\sigma}^*$, i.e. when the actual material strength is higher than it follows from Eqn 5, the strength condition is unactive and the optimum design variables are

$$H = 0.97\left[\beta^2\left(\frac{\rho_c}{\rho_h}\right)^3 \frac{P^4R^2}{E_hE_c^3}\right]^{1/10} \qquad \mathrm{tg}\,\varphi = \frac{1}{2} \qquad \varphi = 26.56^0$$

$$\frac{\delta_h}{a_h} = 0.365\left[\frac{\rho_h}{\beta^4\rho_c}\frac{P^2E_c}{R^4E_h^3}\right]^{1/10} \qquad \frac{\delta_c}{a_c} = \frac{\rho_h\delta_h}{2\rho_c a_c} \tag{7}$$

Parameter β in Eqns 5,6,7 has the following form

$$\beta = \left(1 + \frac{\pi^2 E_h \delta_h^2}{12 a_h^2 G_h}\sin^2 2\varphi\right)^{-1} \tag{8}$$

and allows for transverse shear of helical ribs (G_h is the shear modulus). This parameter is taken into account by an iteration method. First β = 0.7 is taken in Eqns 6 or 7, then structural parameters are found from these equations and substituted in Eqn 8 to find a new value of β and etc. For real carbon-epoxy lattice structures, Eqns 7 usually take place. As can be seen, optimal parameters do not depend on the material strength. However, it is valid only for the foregoing preliminary design.

Introducing normalized mass and force as

$$m = \frac{M}{\pi R^2 L} \qquad p = \frac{P}{\pi R^2}$$

we can write Eqn 4 in the form that does not include the shell dimensions

$$m = 2.67\rho_h\left(\frac{\rho_c}{\beta\rho_h E_h^2 E_c}\right)^{1/5} p^{3/5} \tag{9}$$

This equation demonstrates extremely high weight efficiency of lattice structures. The comparative results for various optimal structures are presented in Fig. 3. Line 1 corresponds to carbon-epoxy stringer shell with a quasi-isotropic skin (reinforced at angles of + 30°, – 30° and 90°). Curve 2 defines the weight efficiency of sandwich shells with light-weight core and quasi-isotropic carbon-epoxy facings, while line 3 is plotted with the aid of Eqn 9. Broken line in Fig. 3 corresponds to traditional stringer-stiffened aluminium shells.

As can be seen, theoretical weight efficiency of a lattice structure is much higher than those for traditional structures. There are several reasons for this effect. First, ribs of the lattice structure provide high membrane and bending stiffnesses of the shell, while their mass is less than the mass of the uniform skin which provides only membrane stiffness and needs to be supplemented with stiffeners or core to yield proper bending stiffness. Second, as was already noted, lattice structures are characterized with high resistance to buckling. And third, unidirectional ribs are loaded in uniaxial compression (helical ribs) and tension (circumferential ribs) for which unidirectional composites demonstrate very high specific strength. Thus, the lattice structure surpass considerably both stringer and sandwich ones in weight efficiency. The curves in Fig. 3 are, naturally, related to ideal structures. Weight efficiency of actual lattice structures which have doors, joints and etc. is significantly lower.

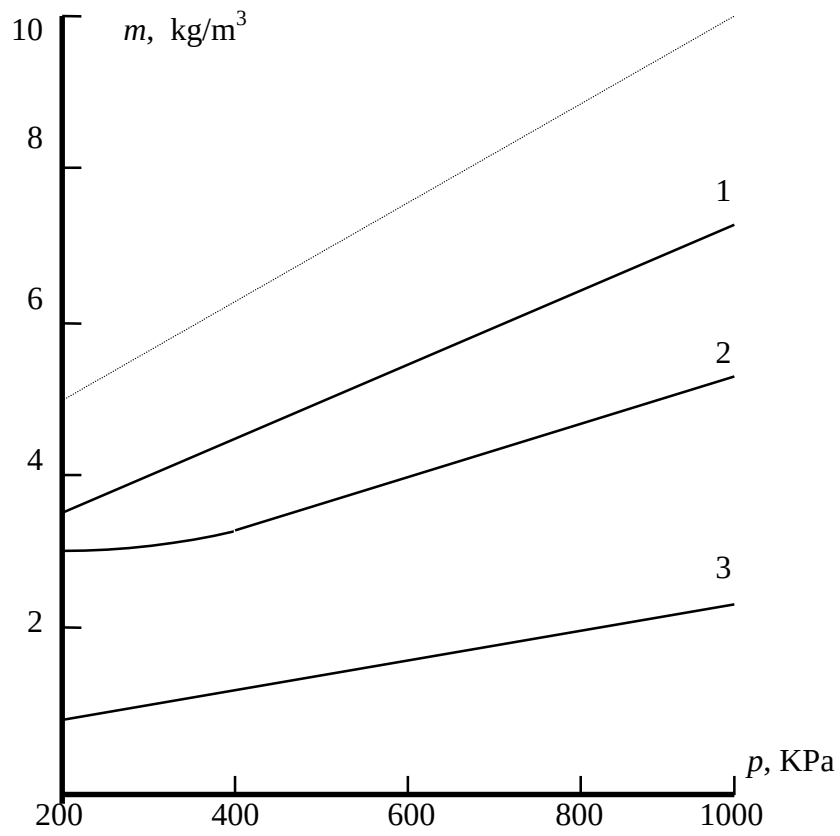

Fig. 3: Efficiency of structures

As can be seen from Eqns 6 and 7, preliminary design specifies only the ratio of rib thicknesses to rib spacings (δ / a). To determine the values of these parameters (δ and a), the refined design based on the finit element and random search methods is undertaken. Refined design is free from the assumptions of the preliminary design the results of which are used as an initial approximation. Refined desin allows us to introduce structural constraints, i.e., to take into account the skin (if the structure requires the skin), doors, joints and to impose manufacturing constraints restricting the number of ribs and their cross-sectional dimensions (e.g., some winding machines require round angular spacings between helical ribs). For real large-scale structures, refined design yields aerial density of the structure as high as (3–6) kg/m^2 depending on the value of the line load, structure of doors and etc.

Manufacturing

Lattice structures are made by automatic winding during which impregnated tows are placed into helical and circumferential grooves formed on the surface of the mandrel. There exist by now several processes to fabricate lattice structures.

1. Free forming of the ribs involves traditional winding process, but tows are placed at a distance from each other on the tops of the tows of the previous layer. This process shown in Fig. 4 is used to manufacture the structures with helical ribs and an outer skin reinforced in circumferential direction. It has the lowest cost but results in poor quality of the ribs.

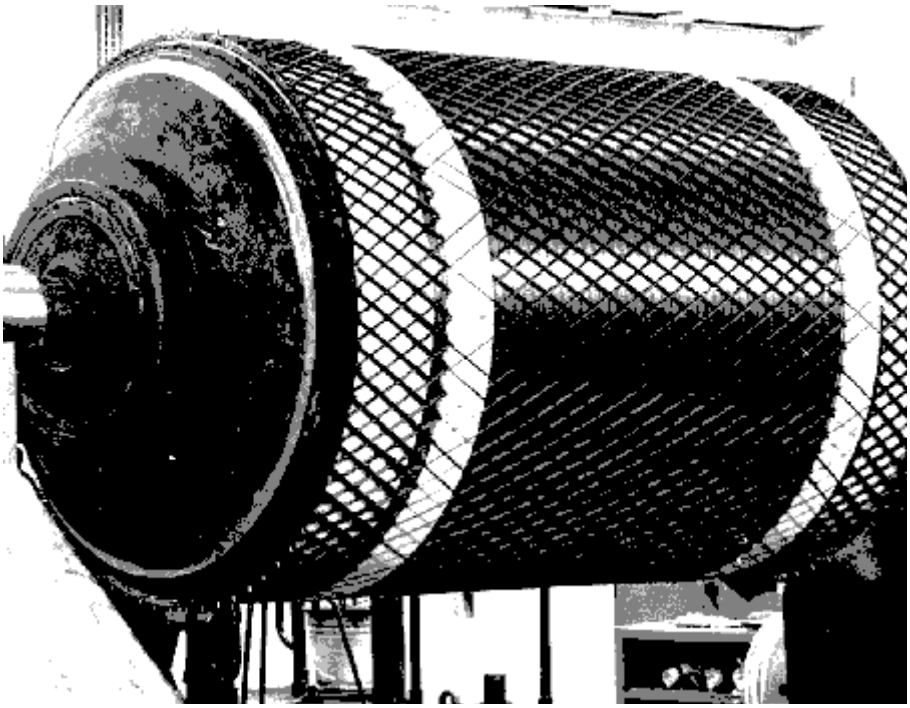

Fig. 4: Winding of helical ribs

2. The low-density foam core with grooves cut for shaping the ribs is applied to the surface of mandrel or an inner skin wound before. After winding and curing the ribs, the foam can be removed or saved. Then the outer skin is wound. This method makes possible to fabricate lattice structures with outer, inner or two-side skin. The structure involving outer and inner skins, rib system and foam combines advantages of lattice and sandwich shells. It is characterized with moderate cost and quality of the ribs, relatively high mass of the lattice structure which can take very high loads and possesses high temperature insolation, damping, and acoustic attenuation.
3. An elastic coating with grooves for shaping the ribs is laid on the mandrel surface. After winding and curing the ribs, the coating is withdrawn into the inner cavity of the shell. This process is used to make lattice structures without skins or with an outer skin. It requires special tools to form the elastic coating but provides proper quality of the ribs and is by now the most widely used process to fabricate lattice structures.
4. A thin metal shell with preformed grooves for ribs is fixed on the mandrel surface. The ribs are wound in the grooves and then an outer skin is formed. Such structure has good tightness and can be used as a load-carrying fuel tank component.

Testing

An important stage in the process of development of lattice structures is the experimental study. The structure of material in structural components is so intricate and dependent on parameters of a specific structure and the corresponding manufacturing process that the determination of mechanical characteristics from specimens or scaled models is not possible. For example, helical ribs have three kinds of alternating material structures with various fiber volume fractions – between rib intersection points, in the vicinity of intersections of helical ribs with each other and with circumferential ribs. The only real way to determine mechanical characteristics of materials supposes making of a special full-size test model, its testing and

subsequent experimental study of its fragments (components of ribs, skin panels, etc.). A considerable difference between obtained results and initial data on the basis of which the structure was designed results in a need for modification of the design and, what is more significant, for a new set of tools. However, the existing experience of fabrication of lattice structures makes it possible to limit the expenses associated with this cumbersome process by the cases involving the development of new structures that are considerably different from existing structures by their dimensions, type of material, joints and doors.

Applications

Existing and possible applications of lattice structures include

- interstages, intertanks, payload adapters, and fairings of launch vehicles,
- aircraft fuselage sections, wing boxes, and ribs,
- helicopter tail beams,
- space telescope bodies,
- submarine bodies,
- masts, columns, pipes and other elements of civil engineering structures.

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