SMA COMPOSITES FOR AERO-STRUCTURE REPAIRS

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SUMMARY: Many current bonded patch repairs are based on a concept that patches bonded on a structure reduce nominal stresses and provide crack bridging in order to reduce stress concentration or range of stress intensity factor and thus extend the service life of the structure. An alternative way to enhance repair efficiency is to apply crack closure stresses, which may be provided by external forces. Shape memory alloy (SMA) has an ability to recover its trained shape as a result of transformation from martensite to austenite when it is heated above a transformation temperature. Consequently, if pre-strained SMA fibres are embedded in a patch to form a SMA composite patch and then placed over the crack, closure stresses may be produced when the SMA fibres are activated. The benefit of the SMA composite patch comes from not only reducing the mean stresses in the cracked plate but also from a possible reduction in the cyclic amplitude of ΔK, or ΔG, relative to the standard patch with the same extensional stiffness. Therefore, this paper is concerned with an application of SMA composite patches on a cracked metallic plate. An upper bound model for the repair of a plate with a crack is presented to illustrate the reduction of energy release rate (G) caused by shape memory effect of SMA fibre reinforced composite patches. Association of the force transfer efficiency with a stress to retard crack growth is described. Some considerations on the design of SMA composite patches are also addressed.

KEYWORDS: shape memory alloy, polymer composites, repairs, SMA fibre reinforced composites, fracture mechanics, energy release rate, closure stresses.

INTRODUCTION

It has been proven [1] that bonded composite repairs display many advantages over mechanical repairs, particularly for damaged aero-structures. Composite bonded repairs generally have no intrusive effect on the repaired structure, provide excellent tailorability and formabilty, possess immunity to corrosion and exhibit good efficiency in reducing stress concentrations. This technique using boron fibre reinforced composite patches has been successfully applied to repair a safety-critical crack on an F111-C lower wing skin [2].

The repair should restore the residual strength of the damaged structure to at least 1.5×DLL (Design Limit-Load) and either prevent further crack growth altogether or else retard damage growth sufficiently to allow for structural integrity management on the basis of safety-by-inspection. Hence, for such a repair of cracked aero-structures, it is crucial to maximise the reduction of stress intensity factor and its range. The current patch repair is based on the concept that patches bonded on structures share external loads, as well as provide crack bridging, in order to reduce the stress concentration or the range of stress intensity factor and so extend the service life of structures. Another effective way to enhance repair efficiency is to apply crack closure stresses. The closure stresses can be produced by residual stresses around the crack tip to increase the fatigue threshold and delay crack growth [3]. Alternatively, externally applied forces can also provide the required closure stresses.

Shape memory alloy (SMA) has the ability to recover its trained shape as a result of transformation from martensite to austenite when it is heated above its transformation temperature. The transformation begins at the austenite start temperature, A_s, and ends at the austenite finish temperature, A_f. Consequently, pre-extended SMA fibres will reach maximum contraction as they are heated above A_f. If they are embedded in composite patches bonded on a cracked area of the structure, a closure stress is produced. It has been found [4] that the shape memory effect in Ti-Ni fibre reinforced epoxy composite can reduce stress concentration and increase fracture toughness of the composite. This paper is concerned with an application of SMA fibre composite patches as an aero-structures repair. The *E*nergy *R*elease *R*ate (ERR) of a crack repaired by SMA fibre reinforced composite patches is to be presented to clarify the reduction of ERR as a result of shape memory effect. Effects of the efficiency of force transfer between the patch and the structure on retarding crack growth will be discussed. Some considerations on the design of SMA patch repairs are subsequently addressed.

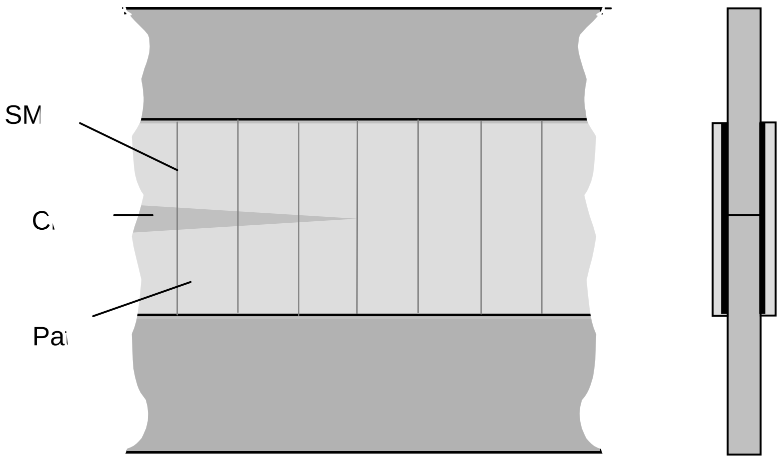

Fig. 1 Schematic description of a cracked structure repaired by SMA fibre reinforced patches.

ERR OF A CRACK REPAIRED BY SMA REINFORCED PATCHES

Following the approach used in [5, 6], it is useful to consider first the case of a two-sided repair, to avoid complications due to out-of-plane secondary bending for a one-sided repair [7]. Accordingly, it is assumed that a cracked plate is repaired by two SMA fibre reinforced composite patches symmetrically bonded on both sides of the plate, and subjected to a remote tensile stress, Σ, as shown in Fig. 1. Effects of the remote stress on the crack can be equivalent to a crack opening stress, σ_0 ($<\Sigma$), along crack surfaces, on the basis of the superposition principle, as shown in Fig.2.

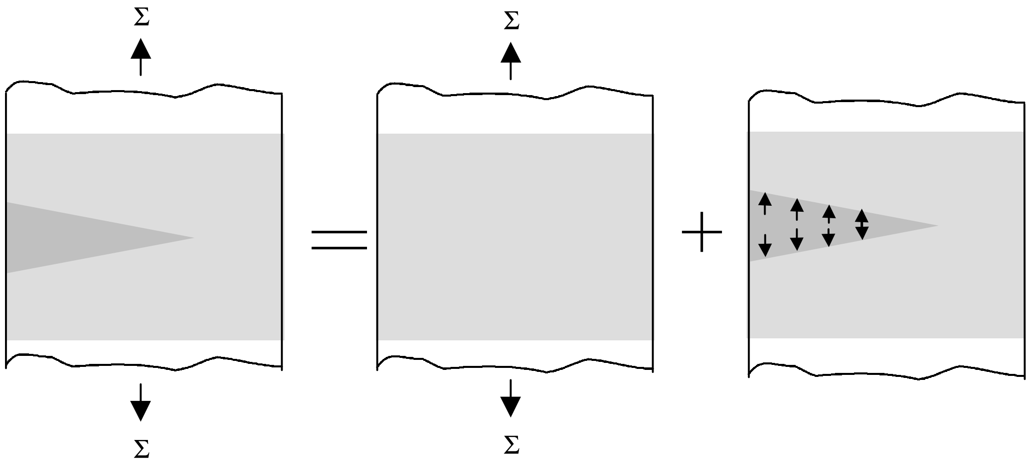

Fig. 2 Superposition for the problem of a crack repaired by SMA reinforced patches.

As indicated by Rose [5], the energy release rate, *G*, of the crack must be less than the minimum value of

- *Case A*: a crack subjected to an open stress σ_0 without bonded patches (see Fig. 3a), and
- *Case B*: an infinite crack subjected to a remote tensile stress Σ with bonded patches (see Fig. 3b).

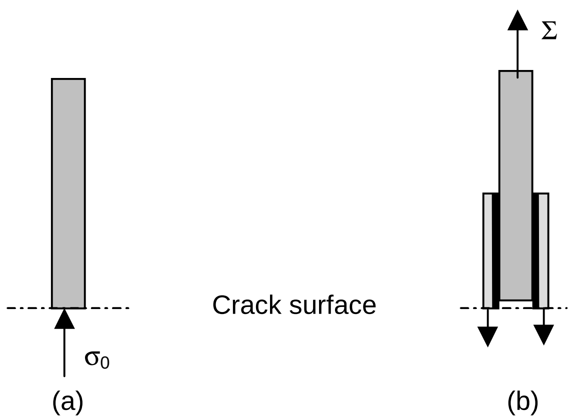

Fig.3 (a) A patch-free cracked plate subject to an opening stress σ_0; (b) a patch-bonded plate with an infinite crack subjected to a remote tensile stress Σ.

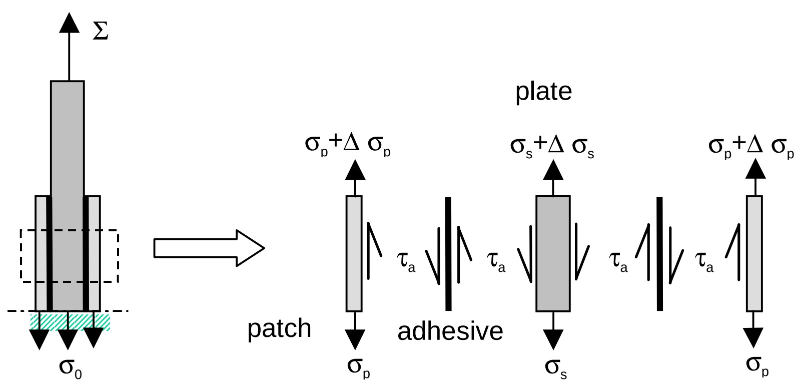

Fig. 4 A description of stresses in different parts of a crack-free plate bonded with patches.

Case A:
As introduced previously, σ_0 can be obtained from a crack-free plate bonded with SMA fibre reinforced patches subject to a remote stress Σ, as shown in Fig. 2. Adhesive layers are assumed to transfer shear stresses only.

As illustrated in Fig. 4, the equilibrium equations are given by:

$$\left.\begin{aligned} \frac{d\sigma_p}{dy} + \frac{\tau_a}{h_p} = 0 \\ \frac{d\sigma_s}{dy} - \frac{\tau_a}{h_s} = 0 \end{aligned}\right\} \tag{1}$$

where σ_p and σ_s are the normal in-plane stress of the patches and plate which have thickness of h_p and $2h_s$, respectively, and τ_a expresses the shear stress transferred through the adhesive layers. Also, the stress-stain relations are expressed by:

$$\left.\begin{aligned} \sigma_p &= E_p\left(\frac{du_p}{dy} + \varepsilon_{SMA}\right) \\ \tau_a &= \frac{G_a}{h_a}(u_s - u_p) \\ \sigma_s &= E_s\frac{du_s}{dy} \end{aligned}\right\} \tag{2}$$

where E_p is the Young's modulus of patches, G_a represents the adhesive shear modulus, ε_{SMA} is the contraction strain induced by shape memory effect of SMA fibres in the patches, and u_p, u_s denote the in-plane displacement of the patches and plate, respectively.

Considering the boundary conditions:

$$\left.\begin{aligned} & u_p\big|_{y=0} = u_s\big|_{y=0} = 0 \\ & \sigma_p\big|_{y=l_p} = 0,\ \sigma_s\big|_{y=l_p} = \Sigma \end{aligned}\right\} \tag{3}$$

the stress, σ_0, and transferred shear stress can be obtained from:

$$\left.\begin{aligned} \sigma_0 &= \frac{1}{\kappa_s + \kappa_p}\left[\left(\kappa_p + \kappa_s/\cosh \Lambda l_p\right)\Sigma - \kappa_s\left(1 - 1/\cosh \Lambda l_p\right)E_s\varepsilon_{SME}\right] \\ \tau_a &= \frac{h_s \Lambda \kappa_s}{\kappa_s + \kappa_p}(\Sigma + E_s\varepsilon_{SME})\frac{\sinh \Lambda y}{\cosh \Lambda l_p} \end{aligned}\right\} \tag{4}$$

where $\kappa_s = 1/E_s h_s$, $\kappa_p = 1/E_p h_p$, $\Lambda^2 = (\kappa_s + \kappa_p)G_a/h_a$, and l_p is the half-width of the patches. As a consequence, for *Case A*, the energy release rate of a crack with length of a in the plate is given by:

$$G_A = \pi\sigma_0^2 a / E_s \quad (5)$$

Case B:

The energy release rate is considered to be the work done to let the normal in-plane stress relax from σ_0 to zero through displacement $u_s|_{y=0}$ at the location of crack surfaces. It is given by:

$$G_B = 2\left(\frac{1}{2}\sigma_0 u_s|_{y=0}\right) \quad (6)$$

where $u_s|_{y=0}$ can be determined from Eq.(1) ~(2) and the boundary conditions,

$$\left.\begin{array}{l} u_p|_{y=0} = 0,\ \sigma_s|_{y=0} = 0 \\ \sigma_p|_{y=l_p} = 0, \sigma_s|_{y=l_p} = \Sigma \end{array}\right\} \quad (7)$$

Hence, we obtain:

$$u_s|_{y=0} = \frac{\kappa_a h_s \Lambda \coth(\Lambda l_p)}{\kappa_s + \kappa_p}\left[\left(\kappa_p + \kappa_s / \cosh \Lambda l_p\right)\Sigma - \kappa_s\left(1 - 1/\cosh \Lambda l_p\right)E_s \varepsilon_{SMA}\right] \quad (8)$$

where $\kappa_a = h_a/G_a$. Substituting Eqs.(4) and (8) into Eq.(6), we have the energy release rate for *Case B*. i.e.:

$$G_B = \Omega\sigma_0^2 / E_s \quad (9)$$

where

$$\Omega = \frac{\Lambda\kappa_a}{\kappa_s}\mathbf{coth(}\Lambda l_p\mathbf{)} \quad (10)$$

Consequently, the energy release rate of a repaired crack satisfies the following inequality,

$$G < \min\left\{\pi\sigma_0^2 a/E, \Omega\sigma_0^2/E\right\}, \quad \sigma_o > 0. \quad (11)$$

The right-hand side term is actually the upper bound of the energy release rate of a crack repaired by SMA fibre reinforced patches. Compared with the case of a standard patch, the benefit of using a SMA composite patch arrises from the reduction in σ_0 due to the term involving the contraction strain ε_{SMA} in Eq. (4). That is, by setting ε_{SMA}= 0 in Eq. (4) one would recover the results for a standard patch. The analysis assumes that the applied stress Σ is sufficient to overcome the compressive stress induced by the patch, i.e. it is assumed that $\sigma_0 > 0$; if $\sigma_0 \le 0$ then G = 0. It is also noted that the benefit comes from not only a reduction in the mean stress, as shown in Fig. 5b, but also from a reduction in the cyclic amplitude of

ΔK, or ΔG, relative to the standard patch with the same extensional stiffness, because the crack is closed when $\sigma_0 < 0$ as shown in Fig. 5c.

The above analysis has ignored residual thermal stresses in the plate due to coefficient of thermal expansion, CTE (α), mismatch between the SMA patch and the aluminium plate ($\Delta\alpha$). In practice the residual thermal stresses will reduce the effectiveness of the SMA patch. If the patch were applied to the plate at a temperature of about 120°C (giving a ΔT of about 100°C) and assuming that the CTE of the aluminium and the SMA are approximately $23\text{x}10^{-6}$ and $10\text{x}10^{-6}$ °C^{-1}, respectively, then a residual tensile thermal strain ($\Delta\alpha\ \Delta T$) of the order of 0.2% would result in the plate. Since recovery strains of the order of 4-8% can be achieved with the SMA then $\varepsilon_{SMA} >> \Delta\alpha\Delta T$, consequently the SMA patch should still be effective in reducing or eliminating crack growth.

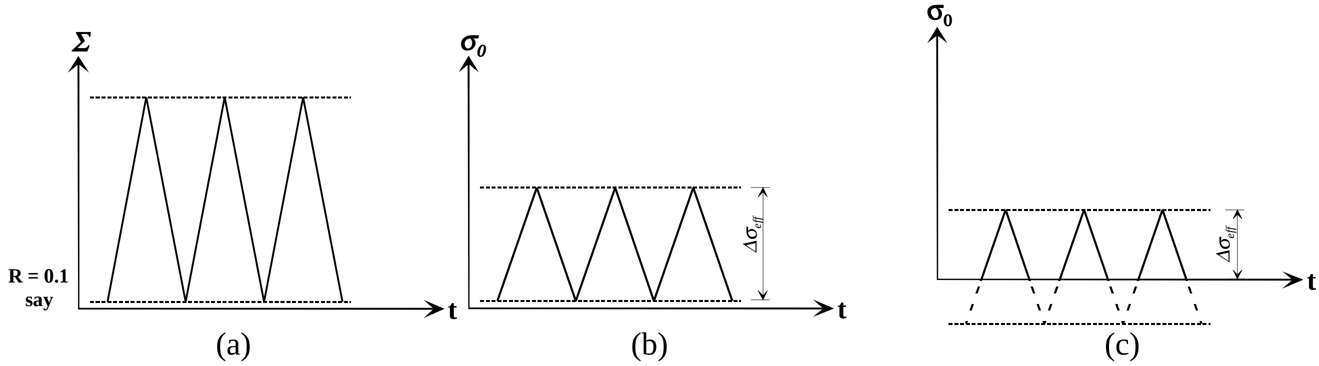

Fig. 5 Time varying stresses on a crack in a plate with (a) no patch, (b) a standard patch and (c) an activated SMA composite patch.

EFFECTS OF FORCE TRANSFER EFFICIENCY ON THE CLOSURE STRESS

The closure stress is now introduced by only considering the term, in Eq.(4), associated with the contraction of SMA fibre reinforced patches. It is expressed by:

$$\sigma_c = -\frac{1}{h_s} f_a \tag{12}$$

where

$$\left.\begin{aligned} f_a &= \int_0^{l_p} \tau_a dy = \frac{h_s\kappa_s}{\kappa_s+\kappa_p}\left(1-1/\cosh\Lambda l_p\right)E_s\varepsilon_{SMA} \\ \tau_a &= \frac{h_s\Lambda\kappa_s}{\kappa_s+\kappa_p}E_s\varepsilon_{SME}\frac{\sinh\Lambda y}{\cosh\Lambda l_p} \end{aligned}\right\} \tag{13}$$

In Eq.(12) and Eq.(13), f_a denotes the shear force transferred between the SMA fibre reinforced patch and the structure through the adhesive layers. It basically contributes a negative value to the opening stress, σ_0, and retards crack growth in the structure.

If the force transfer efficiency is defined by:

$$\eta = \frac{f_a}{f_{\mathbf{max}}} \tag{14}$$

where $f_{\max} = h_p E_p \varepsilon_{SMA}$, called the block force, implying the maximum value which may be produced in patches as their both ends are blocked, the efficiency is obtain as:

$$\eta = \frac{\kappa_p}{\kappa_s + \kappa_p}\left(1 - 1/\mathbf{cosh}\,\Lambda l_p\right) \tag{15}$$

and then the closure stress, σ_c, may be recast as:

$$\sigma_c = -\eta\sigma_{\mathbf{max}} \tag{16}$$

where σ_{max} indicates the maximum closure stress which may be produced by a block force. Obviously, the maximum closure stress and the shear force transfer efficiency determine the closure stress, which is crucial to reducing the crack opening stress.

CONSIDERATIONS ON DESIGN OF SMA PATCH REPAIRS

As discussed previously, the closure stress and the efficiency of force transfer between patches and cracked structures play a very important role in reducing the energy release rate and retarding crack growth. Optimisation of the force transfer efficiency becomes pivotal in designs of SMA patch repairs.

Regarding the force transfer efficiency, Eq.(15) can be rewritten as:

$$\eta = \frac{\kappa_p/\kappa_s}{1+\kappa_p/\kappa_s}\left(1 - \frac{1}{\mathbf{cosh}\left(\sqrt{\dfrac{1+\kappa_p/\kappa_s}{\kappa_a/\kappa_s}}\,l_p\right)}\right) \tag{17}$$

Since

$$\begin{aligned} d\eta/d(\kappa_a/\kappa_s)\big|_{\kappa_p/\kappa_s=constant} &< 0 \\ d\eta/d(\kappa_p/\kappa_s)\big|_{\kappa_a/\kappa_s=constant} &> 0 \end{aligned} \tag{18}$$

the decrease of κ_a/κ_s or increase of κ_p/κ_s may be beneficial to the force transfer efficiency. As a consequence, design of SMA patch repairs should consider: (1) using adhesives with a higher shear modulus and smaller thickness; and (2) selecting SMA fibre reinforced patches with a proper modulus to obtain a higher block force, simultaneously not to reduce the force transfer efficiency by too much. The optimal κ_p/κ_s is determined by

$$(1+\frac{\kappa_p}{\kappa_s})\Lambda l_p\,\mathbf{sinh}(\Lambda l_p) - 2\left[1 - 1/\mathbf{cosh}(\Lambda l_p)\right] = 0 \tag{19}$$

in order to satisfy $d\eta/d(\kappa_p/\kappa_s)=0$.

It should be pointed out that an appropriate design to maximise the efficiency also requires careful consideration about design of the patch itself. It is necessary to understand the effects of matrix properties, SMA fibre lay-up and its volume fraction in patches. As an example, SMA fibre reinforced patches are described in Fig. 6, bonded on the upper and lower surfaces of an aluminium beam. Their material properties are given in Table 1.

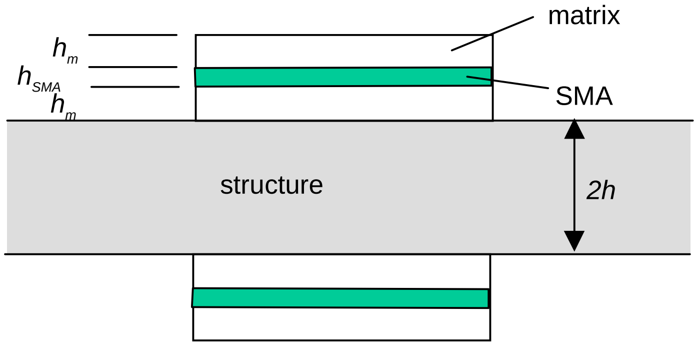

Fig. 6 Schematic description of SMA fibre reinforced patches bonded on a beam.

Table 1 Material properties of patches and structures

	Matrix	SMA	Aluminium
Young's Modulus (GPa)	0.05 ~144	55	72
Poisson's Ratio	0.35	0.30	0.33

A relation between the force transfer efficiency and the matrix modulus is obtained by finite element method analysis. As shown in Fig. 7, the maximum force transfer efficiency can be reached by a proper selection of the modulus. But high matrix moduli are not desirable. Matrix thickness has little influence on the maximum efficiency, but it reduces the efficiency more rapidly as the matrix modulus becomes higher or lower.

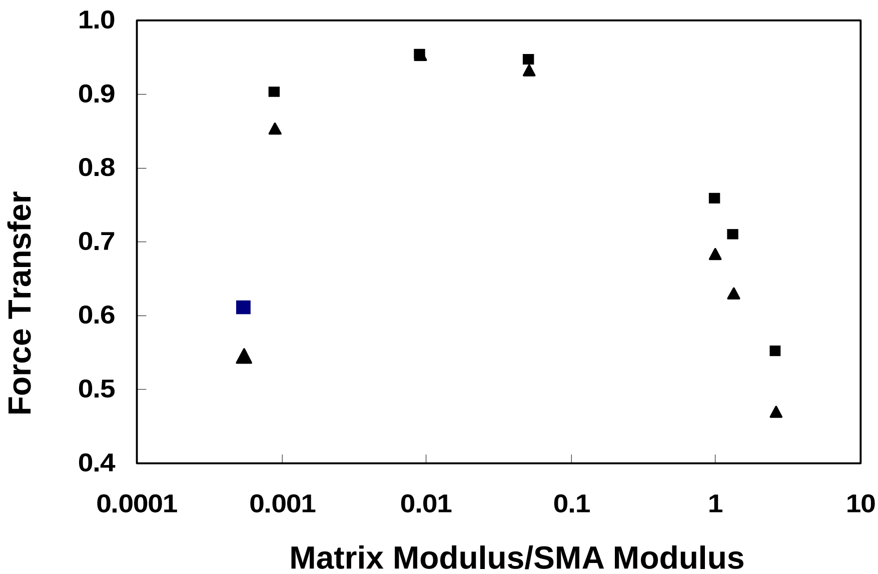

Fig. 7 Variation of force transfer efficiency with increasing matrix modulus / SMA modulus ratio for different matrix thicknesses.

The objective in this section has been to consider patch design from the viewpoint of maximising patch efficiency. However an alternative approach is to consider patch design from the viewpoint of maximising the force applied by the SMA patch on the plate, i.e. to maximise f_a. Therefore future work will entail obtaining an expression for optimal κ_p/κ_s in order to maximise f_a.

CONCLUSIONS

The paper is concerned with the application of SMA fibre reinforced composite patches in aero-structure repair. An upper bound of the energy release rate for a crack repaired by patches is calculated and shown to be reduced by the contraction caused by the shape memory effect. It is also shown that the increase of the contraction strain diminishes the crack opening stress, which may become negative if the contraction strain is large enough. Therefore, the repair does increase the residual strength and extend the durability of the structure.

Further study shows that the crack closure stress greatly relies on the shear force transfer between the patches and the structure, and the force transfer efficiency and the maximum closure stress determine the reduction of the crack opening stress. Moreover, it suggests that repairs should use adhesives with a higher shear modulus and smaller thickness, as well as select a proper modulus of patches to obtain a higher block force without reducing the force transfer efficiency by too much. Furthermore, matrix properties, SMA fibre lay-up and fibre volume fraction in the patches all affect the force transfer efficiency. They should therefore be carefully considered in the design of the patches.

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