MODELING OF THE ELECTROMAGNETIC BEHAVIOUR OF MULTI-LAYER CARBON/EPOXY MATERIALS.

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SUMMARY: An analytical formulation is presented for the computation of scattering and transmission by a non magnetic carbon-epoxy composite material, which is characterized by a complex permittivity. This method employs a first-order state-vector differential equation representation of Maxwell's equations. The solution is given in terms of a 4X4 to transition matrix relating the tangential field components at the input and output planes of the anisotropic region. The complete diffraction problem is solved by combining impedance boundary conditions at these interfaces with the transition matrix relationship.

KEYWORDS: Carbon Fiber Reinforced Composites, Carbon-Epoxy, Electromagnetics, Anisotropy, 4X4-transition Matrix.

INTRODUCTION

Aircraft structures made of Carbon Fiber Reinforced Composites (CFRC) are sensitive to in service impact damage. If the damage is of sufficient importance, strength and service durability of the structure are degraded. The fact that size and location of the damage may only be predictable by statistical means, which is leading to rather conservative designs.

Health Monitoring System (HMS) appear to be quite attractive for such a monitoring, and in the past ten years considerable work has indeed been conducted so as to develop optic fiber strain and damage sensing techniques for composites, in parallel with investigations of compliance change,

acoustic emission and acoustic injection techniques. Such approaches involve using discrete sensors, which are embedded in the composite laminate.

However, many of the difficulties that are associated with the use of such discrete sensors may be overcome by adopting techniques which rely on changes in physical properties of the composite as a consequence of a damage. A good candidate is the so-called electromagnetic technique which is based on changes in permittivity and/or conductivity in the laminate to characterize the damage.

Since 1996 ONERA has been developing such a technique in order to localize defects affecting carbon/epoxy materials. In a first step, this effort has led to the realization of a probe which is able to detect the main defects affecting a carbon/epoxy structure [1].In a second step the development of an integrated system based on the same principle is in progress. But an appropriate modelling of the electromagnetic behavior of multi-layer carbon/epoxy materials should enable to optimize this method and to integrate a sensor network in structure in order to design what would be an electromagnetic HMS. This modeling is considered herein, and as a first step, limited to a structure without defects.

MODELING

Consider the electromagnetic problem in the case where plane waves are obliquely incident upon the *N*-ply laminated carbon-epoxy composites as shown in Fig. 1. Each individual lamina is regarded as a homogeneous and anisotropic sheet. Electrical parameters of the nth ply are described by permeability μ_0 and anisotropic complex permittivity $\bar{\varepsilon}_n$. With respect to the composite principal coordinates (Fig. 2), the permittivity tensor $\bar{\varepsilon}_n$ can be expressed as :

$$\bar{\varepsilon}_n = \begin{bmatrix} \varepsilon_{x'x'} & 0 & 0 \\ 0 & \varepsilon_{y'y'} & 0 \\ 0 & 0 & \varepsilon_{z'z'} \end{bmatrix}_n$$

$$\varepsilon_{qq} = \varepsilon_q + \frac{j\sigma_q}{\omega} \qquad (q = x', y', z')$$

where ε_q is the permittivity, σ_q is the conductivity, and ω is the angular frequency (time-dependence $e^{j\omega t}$ is dropped from now on).

Electromagnetic properties of a uniaxial dielectric material at given point are determined in our case by the fiber direction : on has $\varepsilon_{//}$ along the axis of the carbon fiber, and $\varepsilon_{\perp}$ in the plane perpendicular to this axis (Fig. 2).

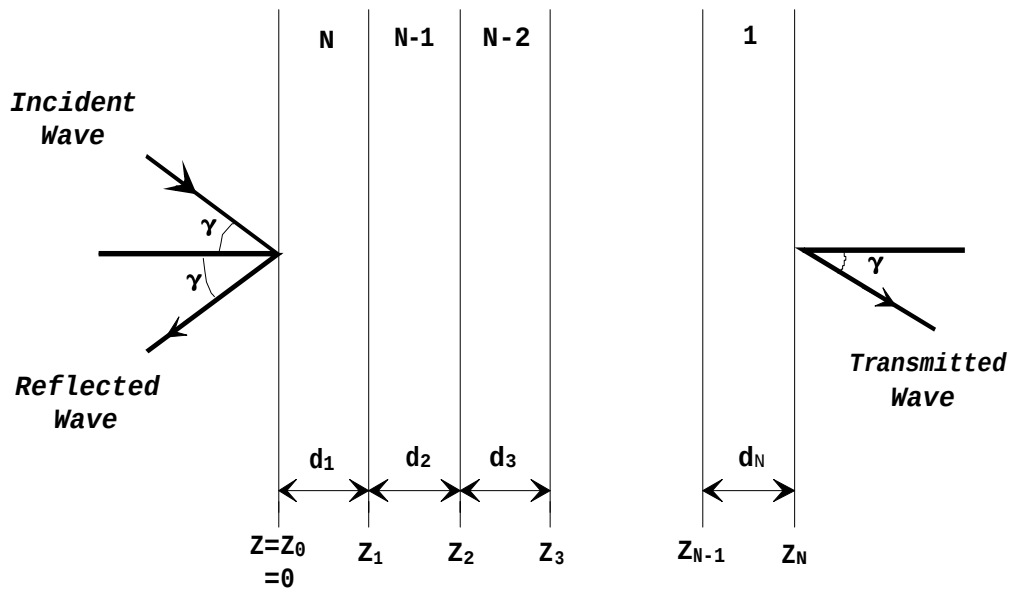

Fig. 1 : *Geometry of N-ply laminated composites.*

By introduction of a rectangular Cartesian coordinate system n_0, y_0, z_0, the direction of the optical axis can be described by the angle φ, and the permittivity tensor in this coordinate system is represented under the form of a symmetric matrix :

$$\bar{\varepsilon}_n = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} = \begin{bmatrix} \varepsilon_s & \varepsilon_{sz} \\ \varepsilon_{zs} & \varepsilon_{zz} \end{bmatrix} \tag{1}$$

with the following elements :

$$\varepsilon_s = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{yx} & \varepsilon_{yy} \end{bmatrix}, \ \varepsilon_{sz} = \begin{bmatrix} \varepsilon_{xz} \\ \varepsilon_{yz} \end{bmatrix}, \ \varepsilon_{zs} = \begin{bmatrix} \varepsilon_{zx} & \varepsilon_{zy} \end{bmatrix} \tag{2}$$

$$\begin{aligned} \varepsilon_{xx} &= \cos^2(\varphi)\ \varepsilon_{//} + \sin^2(\varphi)\ \varepsilon_{\perp} \\ \varepsilon_{yy} &= \sin^2(\varphi)\ \varepsilon_{//} + \cos^2(\varphi)\ \varepsilon_{\perp} \\ \varepsilon_{zz} &= \varepsilon_{\perp} \\ \varepsilon_{xy} &= \varepsilon_{yx} = \cos(\varphi)\ \sin(\varphi)\ (\varepsilon_{\perp} - \varepsilon_{//}) \\ \varepsilon_{xz} &= \varepsilon_{yz} = \varepsilon_{zx} = \varepsilon_{zy} = 0 \end{aligned} \tag{3}$$

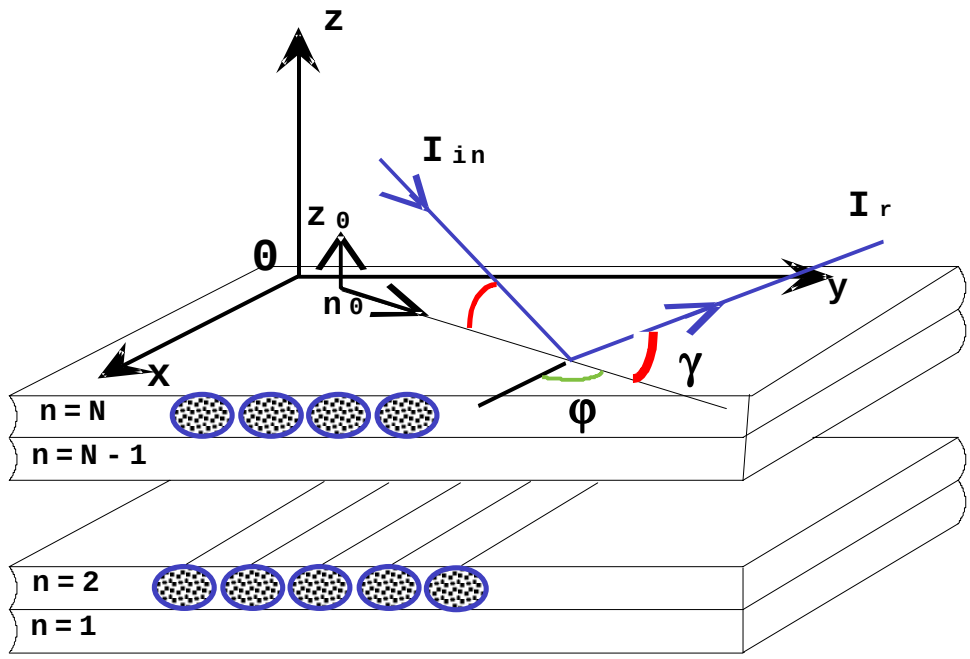

Fig. 2 : *Global coordinates and principal coordinates of laminated composites.*

The electromagnetic field which develops in the plate satisfies the following equations.

The incident plane wave :

$$\mathbf{E}_{in}(\mathbf{r}) = \mathbf{e}_{in}\ \exp(ik_0\mathbf{I_{in}}.\mathbf{r}) \qquad \mathbf{H}_{in}(\mathbf{r}) = \mathbf{h}_{in}\ \exp(ik_0\mathbf{I_{in}}.\mathbf{r}) \tag{4}$$

is impinging upon the plate from the free half-space z > 0 in the direction of the unit vector $\mathbf{I}_{in}$, which is specified by the Bragg angle γ and the azimuthal angle φ.

In the first step a single lamina is considered. Maxwell's equations are projected onto the plane of the plate (x, y) and onto the z-axis, four equations are obtained :

$$\nabla_s \times \mathbf{E_s} = i\,\omega\,\mu_0\ \mathbf{H_z} \tag{5}$$

$$\frac{\partial}{\partial z}\mathbf{z} \times \mathbf{E_s} = i\,\omega\,\mu_0\ \mathbf{H_s} - \nabla_s \times \mathbf{E_z} \tag{6}$$

and

$$\nabla_s \times \mathbf{H_s} = -i\,\omega\,\varepsilon_z\ \mathbf{E_z} \tag{7}$$

$$\frac{\partial}{\partial z}\mathbf{z} \times \mathbf{H_s} = -i\,\omega\,\varepsilon_s\ \mathbf{E_s} - \nabla_s \times \mathbf{H_z} \tag{8}$$

where $\varepsilon_z = \varepsilon_\perp$.

After some transformations of Eqn 5, Eqn 6, Eqn 7 and Eqn 8 a system of two linear algebraic equations obtained where the only unknowns are $\mathbf{E}_s$ and $\mathbf{H}_s$.

$$\begin{cases} \frac{\partial}{\partial z}\mathbf{z}\times\mathbf{H}_s = -i\,\omega\,\varepsilon_s\,\mathbf{E_s} - \nabla_s\times(\frac{1}{i\,\omega\,\mu_0}\nabla_s\times\mathbf{E_s}) \\ \frac{\partial}{\partial z}\mathbf{z}\times\mathbf{E}_s = i\,\omega\,\mu_0\,\mathbf{H_s} - \nabla_s\times(\frac{-1}{i\,\omega\,\varepsilon_z}\nabla_s\times\mathbf{H_s}) \end{cases} \tag{9}$$

Some simple transformations allow to define the following matrix equation :

$$\frac{\partial}{\partial z}\begin{bmatrix}\mathbf{E_s}\\ \mathbf{H_s}\end{bmatrix} = i\,\overline{A}\begin{bmatrix}\mathbf{E_s}\\ \mathbf{H_s}\end{bmatrix} \tag{10}$$

where $\overline{A}$ is a 4X4 matrix with the following elements :

$$A_{11} = A_{12} = A_{21} = A_{22} = 0 \tag{11-a}$$

$$A_{33} = A_{34} = A_{43} = A_{44} = 0 \tag{11-b}$$

$$A_{31} = -\omega\,\cos(\theta)\,\sin(\theta)\,(\varepsilon_\perp - \varepsilon_{//}) - k_x\,k_y\,\frac{1}{\omega\,\mu_0} \tag{11-c}$$

$$A_{32} = -\omega\,(\sin^2(\theta)\,\varepsilon_{//} + \cos^2(\theta)\,\varepsilon_\perp) + k_x^2\,\frac{1}{\omega\,\mu_0} \tag{11-d}$$

$$A_{41} = \omega\,(\cos^2(\theta)\,\varepsilon_{//} + \sin^2(\theta)\,\varepsilon_\perp) + k_y^2\,\frac{1}{\omega\,\mu_0} \tag{11-e}$$

$$A_{42} = -\omega\,\cos(\theta)\,\sin(\theta)\,(\varepsilon_\perp - \varepsilon_{//}) + k_x\,k_y\,\frac{1}{\omega\,\mu_0} \tag{11-f}$$

$$A_{13} = k_x\,k_y\,\frac{1}{\varepsilon_\perp\,\omega} \qquad A_{14} = \omega\,\mu_0 - k_x^2\,\frac{1}{\varepsilon_\perp\,\omega} \tag{11-g}$$

$$A_{23} = -\omega\,\mu_0 + k_y^2\,\frac{1}{\varepsilon_\perp\,\omega} \qquad A_{13} = -k_x\,k_y\,\frac{1}{\varepsilon_\perp\,\omega} \tag{11-h}$$

where k_x and k_y are the horizontal components of the wave vector. Vertical components of waves vector of fields which are propagating in the lamina are the following eigenvalues of the matrix $\overline{A}$ [6] :

$$k_{z1} = -k_{z3} = \sqrt{-\mu_0\,\omega^2\,\varepsilon_{//} + \frac{\varepsilon_{//}}{\varepsilon_\perp}\,k_x^2 + k_y^2} \tag{11}$$

$$k_{z2} = -k_{z4} = \sqrt{-\mu_0\ \omega^2\ \varepsilon_\perp +\ k_x^2 + k_y^2} \tag{12}$$

In Eqn 10, the quantities $\theta, \varepsilon_{//}$, $\varepsilon_\perp$ and the other elements of the matrix $\overline{A}$ take constant values. We can use Eqn 10 to establish the following connection between the values of the matrix function $\begin{bmatrix} \mathbf{E_s} \\ \mathbf{H_s} \end{bmatrix}$ on the upper boundary and lower boundary of layer :

$$\begin{bmatrix} \mathbf{E}_s(z) \\ \mathbf{H}_s(z) \end{bmatrix} = \exp(i\,\overline{A}\ z) \begin{bmatrix} \mathbf{E}_s(z) \\ \mathbf{H}_s(z) \end{bmatrix} \tag{13}$$

Using the Hamilton-Kelly theorem [4, 5], an explicit expression is obtained for the transfer matrix in the form of a finite sum of the four first powers of the matrix $\overline{A}$, i.e. ,

$$\exp(i\,\overline{A}\ z) = \overline{P} = \alpha_1\ \overline{I} + \alpha_2\ \overline{A} + \alpha_3\ \overline{A}^2 + \alpha_4\ \overline{A}^3 \tag{14}$$

where $\overline{I}$ is a 4x4 unitary matrix, and the coefficients σ_1, σ_2, σ_3 and σ_4 are determined by the matrix relation as follow:

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix} = \begin{bmatrix} 1 & k_{z1} & k_{z1}{}^2 & k_{z1}{}^3 \\ 1 & k_{z2} & k_{z2}{}^2 & k_{z2}{}^3 \\ 1 & k_{z3} & k_{z3}{}^2 & k_{z3}{}^3 \\ 1 & k_{z4} & k_{z4}{}^2 & k_{z4}{}^4 \end{bmatrix}^{-1} \begin{bmatrix} \exp(i\,k_{z1}\,z) \\ \exp(i\,k_{z2}\,z) \\ \exp(i\,k_{z3}\,z) \\ \exp(i\,k_{z4}\,z) \end{bmatrix}$$

Now, the whole laminated composite can be considered multilayered anisotropic plate consisting of a set of N uniform layers of a uniaxial dielectric material located between the planes z = 0 and z = -h. Each layer is characterized by the main values of permittivity, the direction of the carbon fibre and the thickness. For convenience, these layers are numbered beginning with the lowest ($n = 1$) and ending with the highest ($n = N$), which borders a free half-space. From the boundary conditions for the electromagnetic field at the interfaces of adjacent layers ($\mathbf{E_s}$ and $\mathbf{H_s}$ should remain continuous across each interface). The following relationship linking values of the matrix function $\begin{bmatrix} \mathbf{E_s} \\ \mathbf{H_s} \end{bmatrix}$ on the upper boundary z = 0 and the lower boundary z = -h of the anisotropic plate is finally obtained :

$$\begin{bmatrix} \mathbf{E_s}(0) \\ \mathbf{H_s}(0) \end{bmatrix} = \overline{P}_{tot} \cdot \begin{bmatrix} \mathbf{E_s}(-h) \\ \mathbf{H_s}(-h) \end{bmatrix} \tag{15}$$

Found in this expression is the joint transfer matrix $\overline{P}_{tot}$ of the anisotropic plate, which is calculated as the product of transfer matrices $\overline{P}_1$, $\overline{P}_2$, … , $\overline{P}_N$, for the individual uniform layers – the first, second, and so on up to the N-th layer,

$$\overline{P}_{tot} = \overline{P}_N . \overline{P}_{N-1} ..... \overline{P}_2 . \overline{P}_1 \tag{16}$$

The transfer matrix $\overline{P_i}$ for each uniform layer i = 1,2,…,N is calculated from Eqn 14.

Each field can be characterized by two components $E_\perp$ and $H_\perp$ which define the components of the wave that are polarized perpendicularly and paralle to the plane of incidence.

The electromagnetic wave for $z = 0$ is the sum of the incident wave being $\begin{bmatrix} E_s^{in} \\ H_s^{in} \end{bmatrix}$ and the reflected wave $\begin{bmatrix} E_s^{r} \\ H_s^{r} \end{bmatrix}$ and for $z = 0$ we have the transmitted wave $\begin{bmatrix} E_s^{tr} \\ H_s^{tr} \end{bmatrix}$. After some transformations we obtain a system of linear algebraic equations whose unknowns are $E_\perp^{r,\,tr}$ and $H_\perp^{r,\,\mathbf{tr}}$.

$$E_\perp^r = R_{ee} \cdot E_\perp^{in} + R_{em} \cdot H_\perp^{in} \qquad \text{(17-a)}$$

$$H_\perp^r = R_{me} \cdot E_\perp^{in} + R_{mm} \cdot H_\perp^{in} \qquad \text{(17-b)}$$

$$E_\perp^{tr} = T_{ee} \cdot E_\perp^{in} + T_{em} \cdot H_\perp^{in} \qquad \text{(18-a)}$$

$$H_\perp^{tr} = T_{me} \cdot E_\perp^{in} + T_{mm} \cdot H_\perp^{in} \qquad \text{(18-b)}$$

With Eqn. 16, the analytical form for R_{ij} and T_{ij} (with i = m, e and j=m, e), is perfectly determined.

CONCLUSIONS

An original analytical model of the electromagnetic behaviour of carbon epoxy multi-layered materials has been elaborated. This model based on a matrix formulation, is easily programmable on a PC computer, in Fortran language. The obtained analytical relationships are in very good accordance with the partially numerical model elaborated in [7].

The experimental validation of this model is in progress. Comparison with measurement of electric and magnetic fields performed on several specimens of carbon epoxy multi-layer plates, illuminated with an electromagnetic plane wave are planned.

This model is part of studies concerning the integrated health monitoring [1]. It has been developed/elaborated in order to detect and characterize the main defects present in carbon epoxy multi-layer structures. These defects are for example delamination, fibre fracture or local burning.

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