

DETERMINATION OF COMPOSITE EFFECTIVE CHARACTERISTICS AND COMPLEX MODULES FOR SHELL STRUCTURES FROM THE SOLUTION OF INVERSE PROBLEM

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SUMMARY: In this paper a numerical - experimental method is proposed to identify the effective mechanical constants for a composite material in shell structures. As experimental data, the results obtained by analyzing the strain state of a material structure under different static loading conditions or the spectrum of natural frequencies and vibration modes are used. Numerical part of the method is based on the solution to the inverse elasticity problem involving the estimation of the model parameters to describe the strain state recorded experimentally. Two models - of elastic anisotropic body subject to static loading and with complex dynamic moduli in the case of vibrations - are considered. In this treatment, concurrent with the solution to the inverse problem, the sensitivity analysis techniques are applied to choose, for example, the required experiments, and the wanted data from a particular experiment. The potential of the method proposed is illustrated numerically.

KEYWORDS: mechanical characteristics of composite, inverse problem, shells

INTRODUCTION

A distinguishing feature of fiber- reinforced composites is simultaneous production of the material and structure by continuous filament framework winding followed by solidification of a polymer matrix. As a result, the material structure is able to vary from point to point which implies the dependence of the structure material properties on the manufacturing technology, the size of an article, the disorientation of fibers, the bending and tension of a framework, and the initial micro-and macroscopic stresses. Therefore, the standard test data on the specimens with a fixed structure or cut from an article workpiece may provide an inadequate assessment of the material behavior within a structure.

One way to define the effective mechanical characteristics without recourse to a reference specimen is to use an experimental evidence for shells under load.

Complex studies in these composite structures require the development of structure-sensitive techniques of determination and control of the material behavior. For this reason, much attention is given today to the identification methods (to the solution of the inverse problem) allowing the refinement of the material characteristics for a particular structure under calculation and the prediction of its behavior in operating regimes.

In this work, we develop further the above-listed investigations by constructing the algorithm for shells of complex geometry under different boundary conditions. The observation is supported by the data on different realistic static loading and on the resonance regimes (the spectrum of natural frequencies and vibration modes). The algorithm should be capable to estimate the test data in the context of their informative value needed to find the required mechanical characteristics.

Mathematical problem formulation

We use here the concept of the inverse problem suggesting that the equation coefficients are determined by the vector

$$\mathbf{a}_p = (a_{p1}, a_{p2}, \dots, a_{pq})^T, \quad (1)$$

where q is the number of the parameters to be found, in particular, the mechanical characteristics which provide the best agreement of the mathematical model for the structure strain state with the experimental observations.

Assume that, for the structure under T^h load, we have the data on the displacement $u_i^{\mathfrak{z}k}$, strains $\varepsilon_{ij}^{\mathfrak{z}k}$ and resonance frequencies $\omega_n^{\mathfrak{z}}$ ($n=1, 2, \dots$). Here, the superscripts \mathfrak{z} and k designate the experimental results at T^h loading

For numerical implementation, a parameterized formulation of the inverse problem is suggested [1, 2], where the vector of coefficients of the state equations \bar{a}_0 , providing within the assumed norm a minimal distance between the calculated $u_i^{rk}, \varepsilon_{ij}^{rk}, \omega_n^r$ and experimental $u_i^{\mathfrak{z}k}, \varepsilon_{ij}^{\mathfrak{z}k}, \omega_n^{\mathfrak{z}}$ results, must be found. As a norm, we take the functional being the sum of mean

$$F(a_p) = \sum_{k=1}^K \int_V \left[\sum_{i=1}^3 \alpha_i^k (u_i^{\mathfrak{z}k} - u_i^{rk})^2 + \sum_{i=1}^3 \sum_{j=1}^3 \beta_{ij}^k (\varepsilon_{ij}^{\mathfrak{z}k} - \varepsilon_{ij}^{rk})^2 \right] dV + \sum_{n=1}^N \gamma_n (\omega_n^{\mathfrak{z}} - \omega_n^r)^2, \quad (2)$$

Where N is the number of natural frequencies known from experiments, and $\alpha_i^k, \beta_{ij}^k, \gamma_n$ are the weight coefficients.

So, we arrive at the functional minimum problem Eqn.(2):

$$F(a_0) = \min F(a_p). \quad (3)$$

provided that the vectors \mathbf{a}_p are in the range of admissible values.

As a rule, the experimental data on the displacement and deformation fields may be obtained at separate points of the region V occupied by the body considered. In this case, functional Eqn.(2) becomes the function of several variables:

$$F(a_p) = \sum_{k=1}^K \left\{ \sum_{m=1}^M \sum_{i=1}^3 \alpha_i^k \left[u_i^{\exists k}(x_m) - u_i^{rk}(x_m) \right]^2 + \sum_{l=1}^L \sum_{i=1}^3 \sum_{j=1}^3 \beta_{ij}^k \left[\varepsilon_{ij}^{\exists k}(x_l) - \varepsilon_{ij}^{rk}(x_l) \right]^2 \right\} + \sum_{n=1}^N \gamma_n (\omega_n^{\exists} - \omega_n^r)^2, \quad (4)$$

and the functional minimum problem reduces to searching for the minimum of the function of several variables. Here, (x_m) and (x_l) are the points for which the experimental data on displacements and deformations are known.

In the general case, oriented composite materials are anisotropic nonlinear viscoelastic materials which, are, however, mostly exploited at temperature below the glass transition temperature. Therefore, in the first approximation these materials can be considered elastic solids, particularly, at small strains, and their mechanical behavior can be described by the model of anisotropic elastic body with elastic constants

$$\sigma_{ij} = C_{ijkl} \varepsilon_{kl} \quad (5)$$

The viscoelastic material behaviour can be described based on the model of viscoelastic body which, for isotropic case, is

$$\sigma_{ij} - \sigma \delta_{ij} = 2G_0 \left\{ \varepsilon_{ij} - \frac{1}{3} \vartheta \delta_{ij} - \int_0^t R(t-\tau) \left[\varepsilon_{ij}(\tau) - \frac{1}{3} \vartheta(\tau) \delta_{ij} \right] d\tau \right\},$$

$$\text{where } \sigma = K_0 \left[\vartheta - \int_0^t T(t-\tau) \vartheta(\tau) d\tau \right]. \quad (6)$$

When the body is subjected to dynamic loading, the use can be made of the material model, where the dissipative properties are described by the complex dynamic moduli.

For isotropic body, the physical equations have the form

$$\sigma_{ij} = K \varepsilon_{ij} \delta_{ij} + 2\bar{G} \left(\varepsilon_{ij} - \frac{1}{3} \vartheta \delta_{ij} \right), \quad (7)$$

Where \bar{G} , \bar{K} are the complex moduli, defined through relaxation kernels R and T [3].

Numerical realization methods of this inverse problem are based on the data obtained in solving the direct static or dynamic deformation problems which involve the following three special problems [3]:

1. Natural vibrations of elastic bodies.

Under uniform boundary conditions, we find the solution of the form:

$$u_i(x, t) = \xi_i(x) \cos \omega t, \quad (8)$$

Where ω is the natural frequency, and $\xi_i(x)$ is the vibration eigenform.

2. Natural damping vibrations of viscoelastic bodies.

Damping vibrations of a viscoelastic body can be represented as

$$u_i(x, t) = e^{-\omega_I t} \left[u_i^c(x) \cos \omega_R t + i u_i^s(x) \sin \omega_R t \right] \quad (9)$$

or in a complex form

$$u_i(x, t) = \bar{u}_i(x) e^{-i\omega t} \quad (10)$$

Where $\bar{u}_i(x)$ are the complex components for the displacement vector amplitudes, $\omega = \omega_R + i\omega_I$ is the complex frequency with the actual part being the frequency and with the imagine part being the damping ratio of natural damping vibrations. In this case, under uniform boundary conditions, we find the solution of the form:

$$u_i(x, t) = \bar{\xi}_i(x) e^{-i\omega t}, \quad (11)$$

Where $\bar{\xi}_i(x)$ is the complex eigenform for shell vibrations.

3. Steady-state forced vibrations.

We find here a periodic in time motion with the period equal the period of external action:

$$u_i(x, t) = \bar{u}_i(x) e^{-ipt}, \quad (12)$$

In the analysis of shell structures, we use the relations from the momentum theory of shells based on the Kirchhoff-Love hypothesis [4].

Our study is concentrated on the shells of revolution. A numerical analysis of represents a FEM semi-analytical variant [5], according to which the components of displacement and loading vectors, and the stress and strain tensors are given in Fourier series along the circumferential coordinate. As finite elements, the element suggested by Zyenkevich O. has been used being the frustum of a cone with the linear approximation (axial and circumferential) and cubic approximation (normal) components of the displacement vector for each harmonic of the Fourier series.

The formulation proposed reduces to classical problem of nonlinear mathematical programming suggesting the minimization of function Eqn.(4) subject to constraints in the form of equalities and inequalities. The last define the range of admissible values of the vector a_p and result from the known constraints imposed on the anisotropic material parameters, see for example [6].

In deciding on a particular method to solve the problem on nonlinear mathematical programming (the optimization method), we take into account, among other factors, the sensitivity of the optimization methods to hindrance (for example, to measurement errors). The methods, applied to construct the non-local approximation of the object function by its values at a number of points (of simplex search type or of barycentric coordinate method type), proved to be least sensitive to measurement errors.

To numerically implement the optimization problem with constraints, we use the simplex method of a sliding admittance based on the Nelder-Mid method [7].

To predict an informative value of the experimental data used to identify the effective material constants and to estimate the possibility of defining any material constants from the same experiment by changing the form of the objective function (for example, on inserting the weight ratios), the sensitivity analysis techniques has been used [2, 8].

Results

Determination of elastic constants in terms of static deformation data for shells

Numerical calculations have been done for cylindrical, cone and semispherical orthotropic shells under different boundary conditions on the shell edges. Static loading has been produced by twisting or tensile forces, and by the internal pressure.

In the numerical investigations made, instead of the full-scale experiments we have normally used the numerical and analytical results obtained at the prescribed values of mechanical characteristics. The calculated displacements and strains have simulated the measurement data from the appropriate experiment whereas the values of mechanical characteristics have served as a reliability criterion for the inverse problem.

The calculation procedure involves the preliminary analysis of the sensitivity coefficients. As would be expected, when the considered shells are loaded by twisting forces, only the sensitivity vector component $\frac{\partial \Psi}{\partial G_{12}}$ differs from zero. Therefore, in solving the inverse problem such data have to provide the way of finding only the modulus G_{12} , as has been calculated. For the analysis, it is more convenient to use a normalized sensitivity vector.

Calculations have been made for different values of the initial approach in the problem on the function minimum (4).

Consideration has been given to the following patterns of shell loading: a) one shell end is fixed and the other is acted upon by the tensile force; b) the shell is subjected to internal pressure, and no stresses are observed on its both edges; c) the shell is subjected to internal pressure, and its both edges are fixed.

By comparing the sensitivity coefficients, we can evaluate the possibility of searching for the corresponding elastic constants. For example, the sensitivity coefficient l_2 , obtained when the shell was loaded according to the pattern a, is small compared to the other coefficients. From the appropriate calculations followed that when the search for the elastic constants was made based on the shell tensile data, the values E_1 and ν_2 could be found sufficiently accurate, and the value E_2 was found provided that a proper choice of the initial approach would be given.

In this problem, the sensitivity analysis techniques can be applied to find the most informative experiments to identify elastic constants. Consider as an example the shell with free stress ends subjected internal pressure. When this shell is reinforced at its midpart by the external rigid ring, the sensitivity coefficients prove to be more commensurable.

Using the spectrum of natural frequency values, we find those frequencies by which the appropriate elastic constants can be determined in terms of the analysis of sensitivity coefficients.

Numerical calculations lead us to the following conclusions.

The experimental data on twisting, extension and internal pressure loading allows us with the aid of the approach proposed to define the elastic characteristics for the orthotropic shells of complex configuration. The results on shell extension can be used not only in solving the optimization problem but in checking calculations.

To identify the mechanical characteristics, which vary linearly along the length of the shell, a set of the experimental data on twisting and internal pressure loading of the shell reinforced with a rigid ring frame can be used.

To find the elastic mechanical characteristics in terms of the natural frequency spectrums, we require the information about one of torsional vibration modes and one or two frequencies of untorsional vibration modes. The last were taken by estimating the sensitivity coefficients. Selected information allows to determinate components of complex dynamic modules.

From the results on modeling the experimental error follows that the data on displacements allow one to find the elastic moduli with an error commensurable with the experimental error, and that for Poisson's ratios is, on the average, twice as large. When the strain values are used as the experimental data, the error arising in defining the elastic constants is less than the measurement error.

To illustrate the above, let us consider the problem of defining the effective elastic constants for a multilayer cylindrical shell obtained by continuous winding. In [9], the experimental data for a shell subjected to internal pressure are given. One end of the shell is fixed, and the other is under tensile force. Two variants of tensile forces were implemented experimentally:

$$T = pR/2 \quad \text{and} \quad T = p(R - R_0)/2.$$

The second corresponds to the case when the nonfixed end cap has the hole of radius R_0 .

The shell geometrical parameters are: $h_i = 0.23$ mm, $R = 100$ mm, $L = 350$ mm, $R_0 = 25$ mm, where h_i is the thickness of a single layer. The shell is composed of 28 layers, of which circular 16 and spiral 6. The winding angle is 29° . The shell is subjected by internal pressure, $p = 0.85$ MPa.

The displacements u_r and u_φ were measured in the four cross-sections. To measure displacements, 6 gauges were mounted on each of these cross-sections. The inverse problem was solved based on the data obtained at the four points along the structure length as the arithmetic mean of the gauge records for the section considered.

The elastic characteristics given in this work for the unidirectional layer ($E_l = 7.10$ MPa, $E_2 = 0.245$ MPa, $\nu_2 = 0.23$, $G_{12} = 0.196$ MPa) allow the calculation of the effective elastic characteristics for the multi-layer shell material by the formulae commonly used to calculate

the mean (effective) elastic characteristics of an arbitrarily reinforced composite, see for example [10].

The differ from the values found by solving the inverse problem is no more than 12 %.

Table 1:

Effective elastic characteristics	E_1 MPa	E_2 MPa	ν_2	G_1 MPa
calculated from the values of parameters of unidirectional layer	$2,012 \times 10^4$	$4,826 \times 10^4$	0,13	$0,7 \times 10^4$
found by solving inverse problem	$1,932 \times 10^4$	$4,216 \times 10^4$	0,127	$0,695 \times 10^4$

Determination of complex dynamic moduli in terms of shell vibration data

We define here the complex dynamic modulus components in the framework of the approach proposed. The body model with the complex dynamic modulus, allows taking into account the material damping properties which can manifest themselves in different ways depending on the motion regime. Under forced vibrations, the dissipative properties show themselves in the values of finite amplitudes (displacements, deformations, and stresses) at the resonance frequencies. The damping properties can be also observed when the free vibrations attenuate in a finite time owing to damping.

To simulate the damping vibration rate, instead of classical problem with initial conditions on free vibrations we suggest a new mechanical spectral problem on natural vibrations in viscoelastic bodies. This problem seems to be more efficient for numerical implementation of the inverse problems.

The first variant of defining the dynamical moduli relates to the use of the experimental data on the amplitude-frequency characteristics of displacements or deformations at some shell points.

The above method is verified based on the numerical results (taken as experimental data) from the solution to the direct problem. The role of the last is played by the problem on steady-state forced vibrations in visco-elastic bodies.

Consider a shell with the geometrical dimensions $L / R = 2$, $h / R = 0.05$. Assume that the mechanical behavior of the material is described by the relations from a linear hereditary theory, and no rheological properties show themselves under volume deformation.

The objective function can be written as

$$F = \sum_{m=1}^M \sum_{j=1}^N \sum_{i=1}^3 \left[u_{ij}^p(z_m) - u_{ij}^3(z_m) \right]^2 \quad (13)$$

Here $u_{ij}^p(z_m)$, $u_{ij}^3(z_m)$ are the calculated and experimental values of displacements at z_m points of the shell at the first N resonance frequencies

Three variants are considered to implement the forced steady-state vibrations of the shell. Figure 1 presents the schemes, which illustrate the boundary disturbing displacements, changing in view of the harmonic law with the amplitude A_0

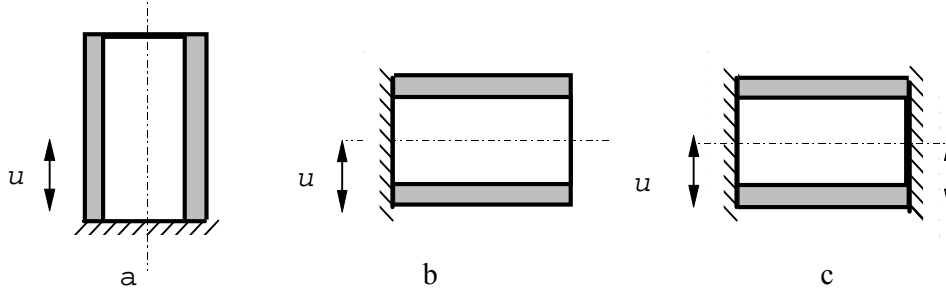


Fig.1: Schematical representation of exciting displacements applied to edges

Amplitude-frequency curves (AFC) for shell displacements and deformations result from the solution to the direct problems simulating the test on a vibration machine.

For calculation, the following model material parameters have been given: $G_R / G_M = 0.2$, $K / G_R = 24.7$, $\rho = 1.0$. Moreover, two types of polyurethane have been considered:

$G_R = 228.02 \text{ kgs / cm}^2$, $G_M = 38.48 \text{ kgs / cm}^2$, $K = 2204.2 \text{ kgs / cm}^2$, $\rho = 0.0001223 \text{ kgs / cm}^3$; and $G_R = 722.41 \text{ kgs / sm}^2$, $G_M = 184.13 \text{ kgs / sm}^2$, $K = 6983.3 \text{ kgs / sm}^2$, $\rho = 0.0001295 \text{ kgs / sm}^3$.

From numerical results it follows that the volume modulus and the complex dynamic modulus components can be found with an accuracy of 0.01 and, for this, we only require the information on amplitude values of displacement or deformation of one or two frequencies at a single harmonic (possibly, non-resonance).

Suppose that we have the test results on time variation of deformation and displacement produced by the initial impulse at different structure points. These data have been treated to distinguish from the general picture the harmonic components which, by virtue of damping, demonstrate attenuation.

When the damping is described by the complex dynamic modulus, every appropriate eigenform can be written as:

$$u(x, t) = \bar{u}(x) e^{-i\omega t} = \bar{u}(x) e^{-i\omega_R t} e^{\omega_I t} \quad (14)$$

The problem on damping natural vibrations in the context of the considered approach applied to solving the inverse problems of material mechanical characteristics can be considered when the objective function has the form:

$$F = \sum_{i=1}^N \left[\left(\omega_{li}^p - \omega_{li}^3 \right)^2 + \left(\omega_{Ri}^p - \omega_{Ri}^3 \right)^2 \right] \quad (15)$$

Calculations have been made based on the experimental data obtained by solving the direct problem the role of which is played here by the problem on damping natural vibrations of a cylindrical viscoelastic shell with one fixed and the other free ends. Dimensions and

properties for the shell material are: $L = 10.6$ cm, $R = 1.65$ cm, $h = 0.2$ cm, $G_R = 228.02$ kgs / cm², $G_M = 38.48$ kgs / cm², $K = 2204.2$ kgs / cm², $\rho = 0.0001223$ kgs / cm³. Complex natural frequencies (sec⁻¹) of the shell vibrations are shown in Table 2.

Table 1. Complex natural frequencies of the shell

harmonic number	frequency number	1	2	3	4
0	ω_I	17.059	28.048	51.283	80.526
	ω_R	203.606	347.099	612.073	974.045
1	ω_I	5.986	23.793	49.379	68.764
	ω_R	73.814	290.223	602.581	835.715

The obtained complex vibration frequencies have simulated the experimental values ω_R^3 and ω_I^3 .

Numerical results showed that the volume modulus and the complex dynamic modulus components can be found with an accuracy of 0.01 and, for this, we only need the information on two natural frequency at a single harmonic. The dependence of volumetric modulus and components of shear dynamic moduli was assumed having sufficiently complex arbitrary form.

CONCLUSIONS

The approach proposed opens the new possibilities for identification of complex dynamic modules of composite materials of shell structures manufactured by continuous filament framework winding.

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