

# ANALYTICAL AND EXPERIMENTAL ELASTIC BEHAVIOR OF TWILL WOVEN LAMINATE

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**SUMMARY:** Woven fabric laminated composites are likely to play a key role in modern lightweight aerospace structures. Before these woven fabric laminated composites can be used in primary structural applications, predictions of elastic properties such as the moduli, Poisson's ratios from the weave architecture and the properties of the constituent materials are desirable. Many models for plain weave, 5- and 8-harness satin weave are available to predict these properties. However, simple analytical model for twill weave pattern is not available. The present paper addresses a basic development of an analytical model to predict the stiffness of the twill composites. Analytical model for twill weave pattern presented in this paper is based on Classical Lamination Theory (CLT). A very simple yet quite general model is developed to obtain the constitutive relationship. The model takes into account effects of the actual fabric structure by considering tow undulations and continuity along both the fill and warp directions. Various tow cross sections are possible and can be easily incorporated for a particular application. The model results in simple formulae to predict in-plane elastic constant, which can be used further in structural analysis.

**KEYWORDS:** Twill, Woven, Textile, Laminate Theory, Elastic Properties

## INTRODUCTION

When Composite materials are considered in structural applications, conventional multidirectional laminates are generally used. Angles of the individual lamina and their stacking sequence in the laminate plate are designed according to the intended loading and geometry of the structure. However, these laminates could be very weak for unintentional loading, especially for the out of plane impact. In these circumstances various alternative composite materials like, textile composites (especially woven) are being developed and tried in place of conventional multidirectional laminates, principally because of good properties in mutually orthogonal directions as well as more balanced properties and better impact resistance than the multidirectional laminates. Textile composites are materials reinforced by a network of tows and are formed by processes such as weaving, braiding or knitting. Woven fabric composites are a two-dimensional class of textile composites where the wrap and fill tows are woven into each other to form a layer. The composite laminate thus formed has good properties in mutually orthogonal directions as well as better out of plane impact resistance than the multidirectional laminate.

Before these woven fabric laminated composites can be used in primary structural applications, predictions of the moduli, Poisson's ratios from the weave architecture and the properties of the constituents are desirable. It is also desirable to have comparison of structural response (e.g. stresses and deformation) of the woven fabric laminated composites with the conventional multidirectional laminates. There are various parameters that characterize the weave architecture of woven laminate composites. Analytical models are necessary to study the effects of these parameters on the behavior of woven fabric composites and to design efficient woven structure for particular application.

The geometry of woven composites is complex which requires complicated 3-D models to predict theoretically accurate mechanical properties. These models in turn require excessively large computations. This may not be practical while designing structures consisting of woven composite laminates. Numerous studies have been done to obtain homogenized macroscopic properties. Chung and Tamma [1] have discussed various homogenization techniques and the bounds on the homogenized macroscopic properties.

The literature review indicates that most of the analytical model development is associated with plain weave, satin weave and eight harness weave architecture. Three different types of models are available in literature: elementary, laminate theory and numerical model [2]. Ishikawa and Chou [3,4] have presented laminate theory models, neglecting two-dimensional extent of the fabric. Here the basic assumption involved is that classical lamination theory (CLT) is valid for every infinitesimal piece of a repeating unit of woven lamina. Usually woven laminates are used in plate like structural forms. Despite the limitations, plate approximation theory offers simple and computationally inexpensive models to predict macroscopic properties. Further work is carried out by Raju and Wang [5], Naik and Shembekar [6], Naik and Ganesh [7] to include two-dimensional extent of the fabric. Whitcomb [8] used 3-D finite element model to analyze plain weave model. All of the work so far discussed deal with plain and satin weave architecture. However, little work is done on twill woven laminates. Scida et al [9] have presented model based on CLT. The weave architecture is described by sinusoidal functions and fiber undulation in both directions is considered.

All the work so far reported, suffer especially two drawbacks. First one is lack of simple closed form formulae which do not require any special computer program. The second one is flexibility of choosing different fiber tow cross sections. Therefore the objective of the current paper is to develop a simple, yet generalized 2-D laminate theory model for twill woven laminate to overcome these shortcomings.

## **OBJECTIVES**

The specific objectives of the present paper are:

- To develop a simple analytical model for twill woven composites using classical laminate theory to predict the extensional stiffness [A]
- To incorporate the effects of tow undulation and continuity along both the warp and fill directions, various cross sections of actual geometry of tow and tow architecture of twill fabric
- To incorporate various tow cross sections which can be chosen to represent actual geometry of the tow.
- To develop simple closed form analytical formulae for twill weave fabric composites which do not require elaborate numerical schemes or any special computer program.

- To compare the results obtained by present analytical model with the experimental results for S2 glass/ C-50 resin material twill fabric composites.

### ANALYSIS OF A MULTIDIRECTIONAL LAMINA

In classical laminate theory for multilayer laminates, the load-deflection behavior of the laminate plate is expressed as:

$$\begin{Bmatrix} \{\mathbf{N}\} \\ \{\mathbf{M}\} \end{Bmatrix} = \begin{bmatrix} [\mathbf{A}] & [\mathbf{B}] \\ [\mathbf{B}] & [\mathbf{D}] \end{bmatrix} \begin{Bmatrix} \{\boldsymbol{\varepsilon}^0\} \\ \{\boldsymbol{\kappa}^0\} \end{Bmatrix} \quad (1)$$

where  $\{\mathbf{N}\}$  and  $\{\mathbf{M}\}$  are stress and moment resultants respectively and  $\{\boldsymbol{\varepsilon}^0\}$  and  $\{\boldsymbol{\kappa}^0\}$  are mid-plane strain and curvature respectively. The resultants  $\{\mathbf{N}\}$  and  $\{\mathbf{M}\}$  are obtained as

$$\begin{Bmatrix} N_x \\ N_y \\ N_s \end{Bmatrix} = \sum_{k=1}^n \int_{h_{k-1}}^{h_k} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_s \end{Bmatrix} dz \quad \text{and} \quad \begin{Bmatrix} M_x \\ M_y \\ M_s \end{Bmatrix} = \sum_{k=1}^n \int_{h_{k-1}}^{h_k} z \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_s \end{Bmatrix} dz \quad (2)$$

where  $n$  is the number of layers and each  $k^{\text{th}}$  layer has bottom and top  $z$ -coordinates  $h_{k-1}$  and  $h_k$ . The contribution of  $\{\boldsymbol{\sigma}\}^k$  of each layer is summed and stresses in  $k^{\text{th}}$  layer  $\{\boldsymbol{\sigma}\}^k$  can be obtained as

$$\{\boldsymbol{\sigma}\}^k = [\mathbf{Q}]^k \{\boldsymbol{\varepsilon}^0\} + z [\mathbf{Q}]^k \{\boldsymbol{\kappa}^0\} \quad (3)$$

where  $[\mathbf{Q}]^k$  is in-plane stiffness matrix of layer  $k$ . For each layer the same  $\{\boldsymbol{\varepsilon}^0\}$  and  $\{\boldsymbol{\kappa}^0\}$  of mid-plane are used to obtain  $\{\boldsymbol{\sigma}\}^k$ . So essentially it is a parallel model. Analysis of composite plate and shell finally reduces to finding  $[\mathbf{A}]$ ,  $[\mathbf{B}]$  and  $[\mathbf{D}]$  matrices for the given geometrical configuration. For multilayer composites these matrices are given as

$$([\mathbf{A}], [\mathbf{B}], [\mathbf{D}]) = \sum_{k=1}^n \int_{h_{k-1}}^{h_k} (1, z, z^2) [\mathbf{Q}]^k dz \quad (4)$$

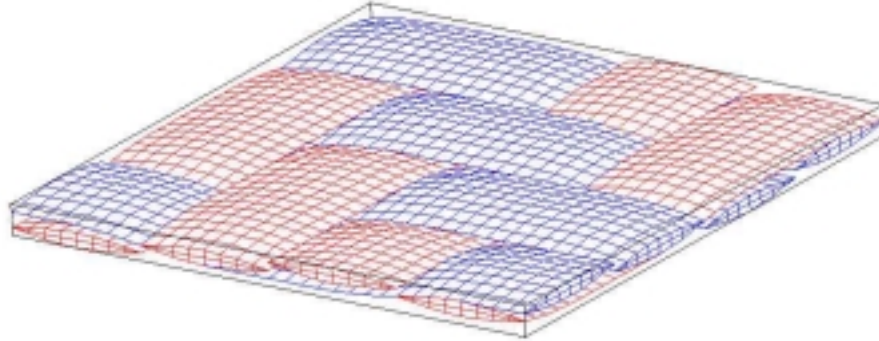
### ANALYSIS OF A TWILL WOVEN LAMINATE

In the present analysis, it is assumed that the Classical Lamination Theory (CLT) is applicable to each infinitesimal piece. The same concept of parallel model, as discussed in the earlier section, can be extended to woven and in particular to twill composites. In multidirectional laminates  $[\mathbf{Q}]^k$  in Eqn. (4) does not vary in any given layer i.e. no material variation in parallel plane. However, this is not the case in woven composites. Because the warp and fill fiber tows are woven into each other to form a layer,  $[\mathbf{Q}]$  varies in all three,  $x$ ,  $y$  and  $z$  directions. Hence Eqn. (4) needs to be modified and can be written as

$$([\mathbf{A}], [\mathbf{B}], [\mathbf{D}]) = \frac{1}{P_A} \iiint_{RU} (1, z, z^2) [\mathbf{Q}] dV \quad (5)$$

where the integration is performed over a repeating pattern called Repeating Unit, RU and  $P_A$  is its area on  $xy$ -plane. The limits of integration are chosen appropriate to the selected repeating unit.

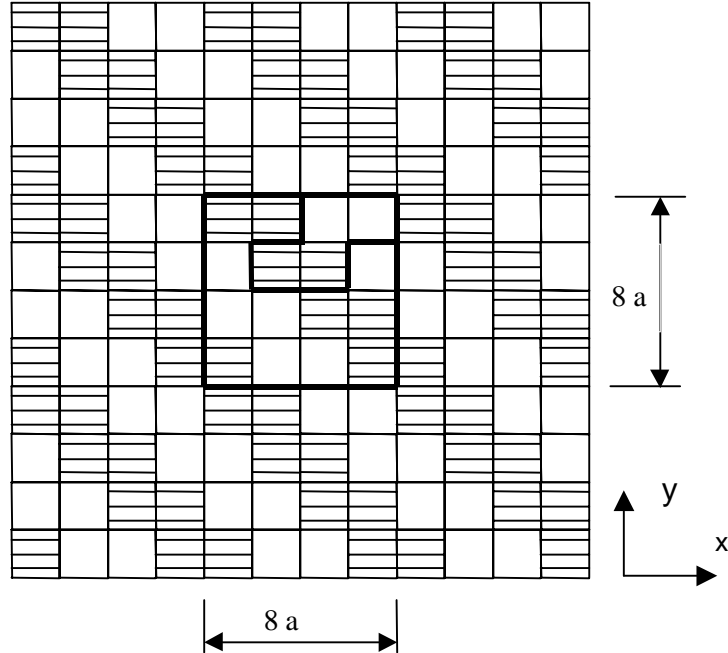
In woven fabric structure fill and warp tows are interlaced (see Fig. 1) over each other with resin (matrix material) filled in the pockets. The  $[Q]$  of matrix material is the same throughout the integration. However,  $[Q]$  in tow region is not constant. So integration in Eqn. (5) has to be performed in different regions with appropriate  $[Q]$  and limits of integration.



*Figure 1. Fill and warp tow interlacing in twill woven laminate*

### **Twill woven fabric – general structure**

In the present case of twill weave pattern, following repeating unit (Fig. 2) is identified. This weave pattern in xy-plane. Tows running along x-axis are called fill tows and those along y-axis are called warp tows. Only squares with fill at the top are shown. The blank square means the warp tow is at the top.



*Figure 2. Repeating unit and integration unit for twill woven structure*

Note that each small square has a dimension  $2a$  and the square repeating unit has dimension of  $8a$ . This repeating unit is also useful of finite element modeling and analysis. This geometrical weaving pattern can be generated from geometry of weave in S shaped region as shown inside the repeating unit in Fig. 2. Integration in this S shape is repeated four times in repeating unit.

This S shaped region is called Integration Unit, IU. The IU is one fourth of RU. So Eqn. (5) becomes

$$([A]) = \frac{1}{16a^2} \iiint_{IU} [Q] dV \quad (6)$$

The cross section and key dimensions of IU are shown in Figs. 3 and 4. The reference xy-plane is same as mid-plane, so that z limits are from  $-2b$  to  $+2b$  as total thickness is  $4b$ . Sections AA and BB of Fig. 4.1 are in xz-plane where as sections CC, DD and EE of Fig. 4.2 are in yz-plane.

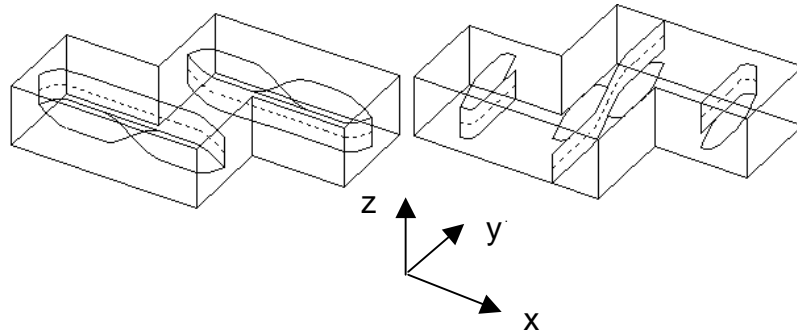


Figure 3. Tow undulation in integration unit of twill woven structure

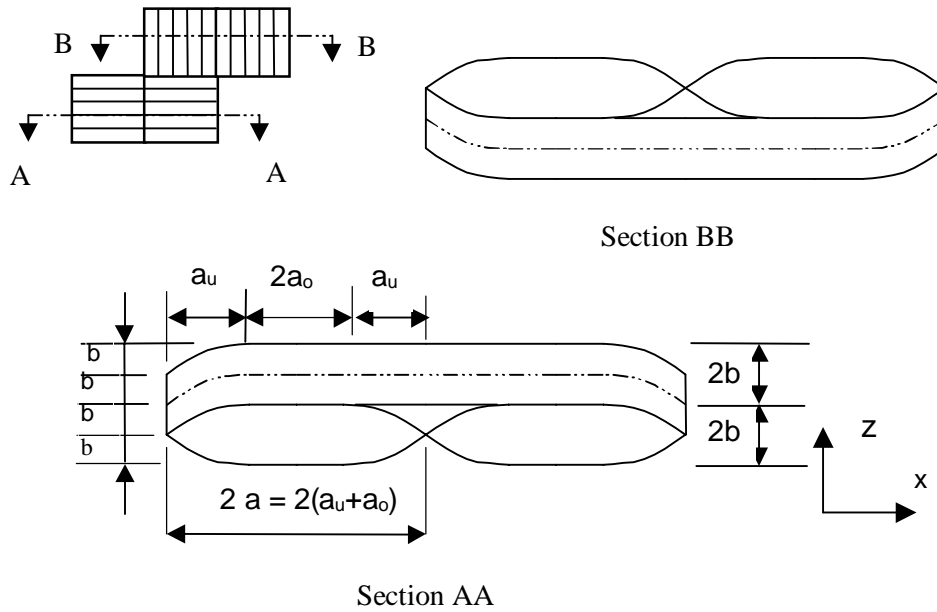


Figure 4.1- Details of tow undulation in integration unit of twill woven structure

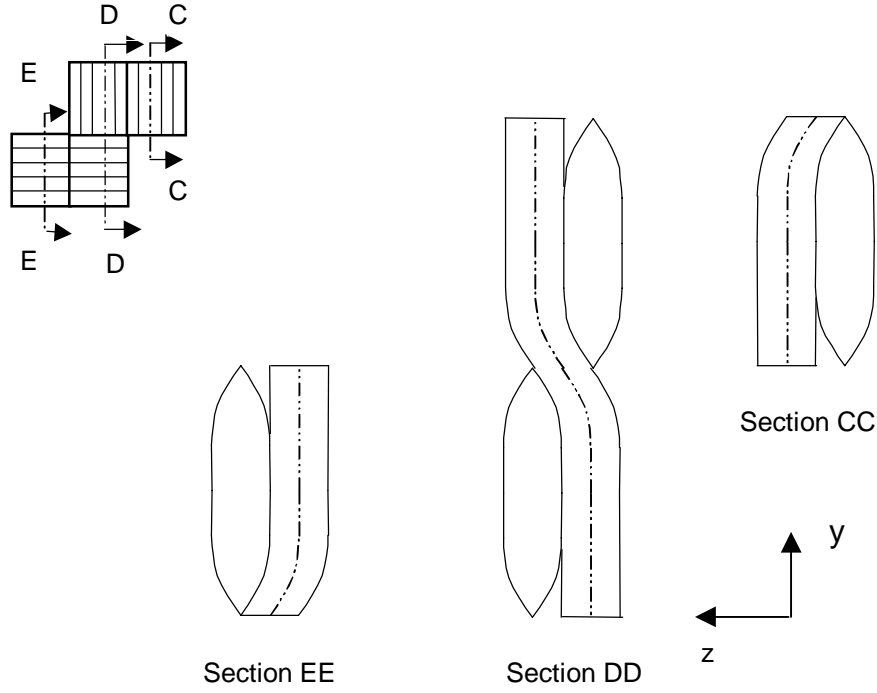


Figure 4.2- Details of tow undulation in integration unit of twill woven structure

### Twill woven fabric – assumptions

Following assumptions are made to facilitate the integration given in Eqn. (6).

1. The current analysis is restricted to non-hybrid twill woven laminate. It means the geometry of tow path and tow cross section is identical for both fill and warp tows.
2. Fill and warp tows are tightly woven over each other. It means if the cross section is taken through the center of tow cross section as shown in Fig. 4, there is no matrix in between fill and warp tows. It also means that if fill and warp tows each have thickness  $2b$ , then total laminate thickness will be  $4b$  as shown in Fig. 4.1. This is further explained in the following Fig. 5.
- 3.

Both have the same equation

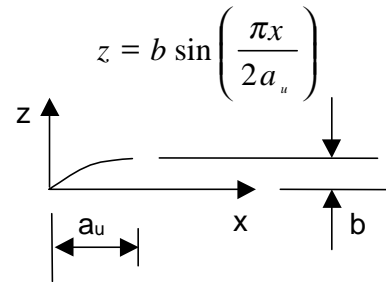
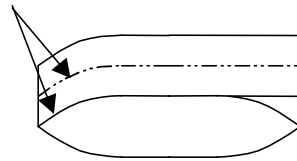


Figure 5. Equation of the tow path and tow cross section.

4. The Eqn. of tow path and the part of the tow cross section as shown in Fig. 5

is given by  $z = b \sin\left(\frac{\pi x}{2a_u}\right)$

5. The orientation of tow cross sections remains constant along the tow path and the tow cross sections are normal to the global axes  $x$  or  $y$ . They are not normal to the undulating tow path. In reality this may not be the case but this assumption immensely simplifies the integration given Eqn. (6).

## APPROACH

Regions and limits of integration are complex in general. However, they can be immensely simplified by the following approach.

$$\iiint [\mathbf{Q}] dV \text{ can be written as } \int ([\mathbf{Q}]) \iint dydz \, dx \text{ or } \int ([\mathbf{Q}]) \iint dxdz \, dy$$

In the case of fill tow regions  $\iint dydz$  is the cross section of the fill tow and in the case of warp tow regions  $\iint dxdz$  is the cross section of the warp tow.  $[\mathbf{Q}]^{\text{Tow}}$  is constant on the tow cross section and thus it can be taken out of integration  $\iint dydz$  and  $\iint dxdz$ . Thus  $\iiint [\mathbf{Q}] dV$  can be in general written for both tow regions as

$$\iiint [\mathbf{Q}]^{\text{Tow}} dV = A \int_{\xi_1}^{\xi_2} [\mathbf{Q}]^{\text{Tow}} d\xi$$

where  $\xi$  is either  $x$  or  $y$  depending on the tow region and  $A$  is the tow cross section along the tow path. Since  $A$  is constant along the path (assumption 4) and thus is taken out of integration. Volumes in IU occupied by matrix material are complex. However, they can be analyzed as some area swept along  $x$  or  $y$  direction. However, unlike tows, these swept areas may be varying along the path of the sweep and  $[\mathbf{Q}]$  is constant. Thus  $\iiint [\mathbf{Q}] dV$  can be written as

$$\iiint [\mathbf{Q}]^M dV = [\mathbf{Q}]^M \int_{\xi_1}^{\xi_2} A(\xi) d\xi.$$

Following this approach the volume in the integration in Eqn. (6) can be divided into two parts: along tow path and remaining resin (matrix pockets). So  $[\mathbf{A}]$  matrix can be expressed as

$$([\mathbf{A}]) = \frac{1}{16a^2} \iiint_{\text{AlongTowPath}} [\mathbf{Q}] dV + \frac{1}{16a^2} \iiint_{\text{MatrixPockets}} [\mathbf{Q}] dV \quad (7)$$

### Tow cross section and volume fraction

Consider a cross section of the fiber tow as shown in Fig. 6.

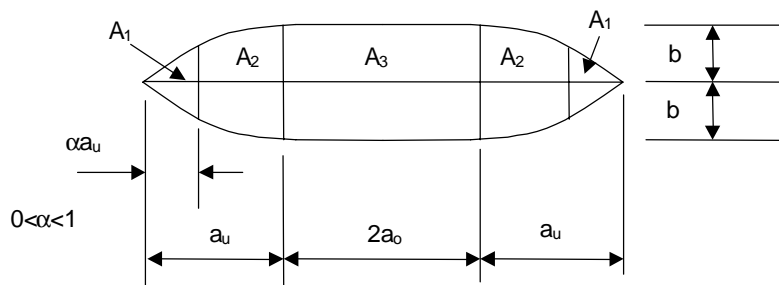


Figure 6. Tow cross section

Area 1 is associated with matrix material.  $\alpha$  is truncation parameter of the tow cross section. Value of  $\alpha$  varies from 0 to 1.

$$A_1 = \frac{8}{\pi} a_u b A_1(\alpha), \text{ where } A_1(\alpha) = 1 - \cos(\pi\alpha/2)$$

$$A_2 = \frac{8}{\pi} a_u b A_2(\alpha), \text{ where } A_2(\alpha) = \cos(\pi\alpha/2) \text{ and } A_3 = 4a_0 b \quad (8)$$

The relationship among the weave parameters ( $\alpha$ ,  $a_u$ ,  $a_o$ ,  $b$ ) can be obtained by considering the volume fraction of fiber in the unit cell,  $V_f$  and packing density of fiber in the tow,  $p_d$ . It can be shown that

$$V_f = \frac{\frac{2}{\pi} a_u \cos\left(\frac{\pi\alpha}{2}\right) + a_o}{a} p_d$$

where  $a = a_u + a_o$  is the half pitch width of the tow. (Fig. 4.1).  $V_f$ ,  $p_d$ ,  $b$  and  $a$  are process and configuration parameters. They are known or given. The rest of the weave parameters i.e.  $\alpha$ ,  $a_u$  and  $a_o$ , depend on them. By performing computation of integration along tows and matrix region and substituting into Eqn. (7),  $[\mathbf{A}]$  matrix can be expressed as

$$[\mathbf{A}] = \frac{4\left(\frac{\pi - 2}{\pi}\right) a_u b + A_1}{(a_0 + a_u)} [\mathbf{Q}]^M + \frac{A_2 + A_3}{4(a_0 + a_u)^2} \int_0^{a_u} ([\mathbf{Q}]_{\theta}^F + [\mathbf{Q}]_{\theta}^W) d\xi$$

$$+ \frac{(2a_0 + a_u)(A_2 + A_3)}{4(a_0 + a_u)^2} ([\mathbf{Q}]_{\theta=0}^F + [\mathbf{Q}]_{\theta=0}^W)$$

## MODEL VALIDATION

Number of tension tests were carried out using S2 glass / C-50 resin coupons. The specimens were mounted with pair of strain gages to measure the axial and transverse strains. Using the experimental data the average values of modulus and Poisson's ratio were obtained and are given in Table 4. There were 45 layers (all  $\theta = 0^\circ$ ) of S2-Glass/ C-50 resin material system. From the photomicrograph the tow cross section was found to be oval shape with no flat portion in it. The measured average weave parameters,  $a_u$  and  $b$  are given in Table 1 and the raw fiber and resin material data is given in Table 2.

*Table 1- Geometrical parameters of the unit cell of the S2-Glass/ C-50 resin material system*

Undulation dimension (Half width of tow)	$a_u$ (mm)	1.18
Half thickness of tow	$b$ (mm)	0.125
Volume fraction of tow in the unit cell (From model geometry)	$V_t$	0.64
Volume fraction of fiber in the laminate (Given by laminate supplier*)	$V_f$	0.5
Packing density of fiber in the tow (Computed from $v_t$ and $v_f$ )	$p_d$	0.83



*Table 2-Fiber and resin material properties*

<b>S2-Glass *</b>		
Longitudinal Young's modulus	$E_{1f}$ (GPa)	85.55
Poisson's ratio	$\nu_{12f}$	0.23
<b>Resin **</b>		
Young's modulus	$E_m$ (GPa)	3.45
Poisson's ratio	$\nu_m$	0.35

\* Daniel and Ishai [10] , \*\* Tuskegee University)

From the fiber and resin properties, given in Table 2, and packing density of fiber in the tow,  $p_d$ , (Computed from  $V_t$  and  $V_f$ ), the effective tow properties were estimated by Composite Cylinder Model (CCA) of Hashin [11] as quoted by Naik and Shembekar [6]. The computed properties are given in Table 3.

*Table 3- Mechanical properties of s2-Glass / C-50 resin material system  
assuming packing density of fibers in tow,  $p_d = 0.83$*

Longitudinal Young's modulus	$E_1$	GPa	71.6
Transverse Young's modulus	$E_2 / E_3$	GPa	23.7
Poisson's ratio	$\nu_{12} / \nu_{13}$	-	0.2197
Poisson's ratio	$\nu_{23}$	-	0.2726
Axial Shear Modulus	$G_{12} / G_{13}$	GPa	10.2
Transverse Shear Modulus	$G_{23}$	GPa	8.4

The comparison of analytical and experimental results is given in Table 4. The table shows reasonably good agreement.

*Table 4- Comparison of analytical and experimental results*

<b>Properties</b>		<b>Analytical</b>	<b>Experiment</b>
Young's modulus	$E_x / E_y$ (GPa)	30.6	28.7
Poisson's ratio	$\nu_{xy}$	0.1327	0.1366
Shear Modulus	$G_{xy}$ (GPa)	6.83	N/A

## CONCLUSIONS

A generalized 2-D model for twill woven laminate is developed to predict elastic material constants. The tow undulation and continuity along both the warp and fill directions are considered. The present analysis results in elegant but simple closed form formulae to obtain extensional stiffness matrix of twill woven laminate. The results obtained by using present analytical model agreed well with those obtained from the experiments. The present analysis can be effectively used in design and analysis of twill woven laminates.

## ACKNOWLEDGEMENT

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