DELAMINATION OF TAPERED COMPOSITE LAMINATES SUBJECTED TO MECHANICAL AND HYGROTHERMAL LOADINGS

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SUMMARY: Tapered laminate is susceptible to delamination due to the singular interlaminar stresses at the end of the terminating ply. The onset and growth of the delamination can be characterized by the potential energy release rate based on the fracture mechanics. The potential energy release rate associated with the delamination growth into the thick section of the tapered laminate under mechanical and hygrothermal loadings are analyzed by the superposition method. Transforming the original tapered section problem with mechanical and hygrothermal loadings to that with mechanical tractions applied on the end surface of the terminating dropped ply, we can clarifiy the effect of the hygrothermal loading on the energy release rate as well as that of mechanical loading.

KEYWORDS: tapered laminate, ply drop, delamination, energy release rate, mechanical loading, hygrothermal loading, superposition method, fracture mechanics.

INTRODUCTION

Taperd sections with dropped plies are commonly used where a change in thickness is required in the composite laminates. Tapered laminate is susceptible to delamination due to development of the singular interlaminar stresses at the ends of the terminating plies. The onset and growth of the delamination can be predicted by the potential energy release rate based on the fracture mechanics. The potential energy release rates for delamination originating from free edge or transverse crack tip were obtained by O'Brien [1, 2] based on the compliance change due to the delamination growth while they were analyzed by Kondo [3] based on the strain energy change in the superposition method. The potential energy release rate for delamianation from the end of the dropped ply of tapered laminate subjected to tensile loading was obtained by Wisnom et. al. [4] based on the compliance change due to the delamination growth.

In the present paper, the superposition method is applied to obtain the potential energy release rate associated with the delamination growth in tapered laminates subjected to mechanical and

hygrothermal loadings. By transforming the original tapered section problem with mechanical and hygrothermal loadings to that with mechanical loading applied by the end surface of the terminating ply, we can easily analyze the effect of the hygrothermal loading on the potential energy release rate for the delamination in tapered laminates as well as that of the mechanical loading.

SUPERPOSITION METHOD

We consider a tapered section of laminate subjected to mechanical and hygrothermal loadings as shown in Fig.1. First, we obtain a uniform stress distribution in an infinite uniform laminate without the thicker part based on the classical lamination theory. Then, we analyze the interlaminar stresses between the terminating dropped ply and the adjacent continuous ply by the superposition method which utilizes the uniform stress distribution for the infinite laminate.

Uniform Stresses in Infinite Laminate

The stress-strain relationship for each lamina in the material coordinates is given by

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \left(\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} - \begin{bmatrix} \varepsilon_{10} \\ \varepsilon_{20} \\ \gamma_{120} \end{bmatrix} \right) \tag{1}$$

where the initial strains due to the temperature change *DT* and the moisture content change *DM* are

$$\begin{bmatrix} \varepsilon_{10} \\ \varepsilon_{20} \\ \gamma_{120} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{bmatrix} \Delta T + \begin{bmatrix} \beta_1 \\ \beta_2 \\ 0 \end{bmatrix} \Delta M \tag{2}$$

which are divided into the isotropic and deviatoric parts as

$$\begin{bmatrix} \varepsilon_{10} \\ \varepsilon_{20} \\ \gamma_{120} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \left(\frac{\alpha_1 + \alpha_2}{2} \Delta T + \frac{\beta_1 + \beta_2}{2} \Delta M \right) + \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \left(\frac{\alpha_1 - \alpha_2}{2} \Delta T + \frac{\beta_1 - \beta_2}{2} \Delta M \right) \tag{3}$$

The stress-strain relationship in the laminate coordinates are derived from Eqn 1 as

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \left(\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} - \begin{bmatrix} \varepsilon_{x0} \\ \varepsilon_{y0} \\ \gamma_{xy0} \end{bmatrix} \right) \tag{4}$$

The generalized stresses are

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \int \begin{bmatrix} \mathbf{I} \\ z\mathbf{I} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} dz \tag{5}$$

We concider an infinite laminate subjected to uniform mechanical loading such that all the generalized stresses are prescribed and uniform hygrothermal loading such that the temperature and moisture content changes are given by

$$\Delta T = \Delta T_0 + \Delta T_1 z \qquad \Delta M = \Delta M_0 + \Delta M_1 z \tag{6}$$

Substituting Eqn 6 into Eq.3 we get

$$\begin{bmatrix} \varepsilon_{10} \\ \varepsilon_{20} \\ \gamma_{120} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} (\Delta\Theta_0 + \Delta\Theta_1 z) + \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} (\Delta\Theta'_0 + \Delta\Theta'_1 z) \tag{7}$$

where

$$\begin{aligned} [\Delta\Theta_0 \quad \Delta\Theta_1] &= \frac{\alpha_1 + \alpha_2}{2} [\Delta T_0 \quad \Delta T_1] + \frac{\beta_1 + \beta_2}{2} [\Delta M_0 \quad \Delta M_1] \\ [\Delta\Theta'_0 \quad \Delta\Theta'_1] &= \frac{\alpha_1 - \alpha_2}{2} [\Delta T_0 \quad \Delta T_1] + \frac{\beta_1 - \beta_2}{2} [\Delta M_0 \quad \Delta M_1] \end{aligned} \tag{8}$$

The initial strain in the laminate coordinates are derived from Eqn 7 as

$$\begin{bmatrix} \varepsilon_{x0} \\ \varepsilon_{y0} \\ \gamma_{xy0} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} (\Delta\Theta_0 + \Delta\Theta_1 z) + \begin{bmatrix} \cos 2\theta \\ -\cos 2\theta \\ 2\sin 2\theta \end{bmatrix} (\Delta\Theta'_0 + \Delta\Theta'_1 z) \tag{9}$$

The strains in the laminate subjected to uniform mechanical and hygrothermal loadings can be expressed in terms of the generalized strains as

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \bar{\varepsilon}_x \\ \bar{\varepsilon}_y \\ \bar{\gamma}_{xy} \end{bmatrix} + z \begin{bmatrix} \bar{\kappa}_x \\ \bar{\kappa}_y \\ \bar{\kappa}_{xy} \end{bmatrix} = [\mathbf{I} \quad z\mathbf{I}] \begin{bmatrix} \bar{\varepsilon}_x \\ \bar{\varepsilon}_y \\ \bar{\gamma}_{xy} \\ \bar{\kappa}_x \\ \bar{\kappa}_y \\ \bar{\kappa}_{xy} \end{bmatrix} \tag{10}$$

which coincide with the Kirchhoff hypothesis employed in the classical laminateion theory.

Substituting Eqn 9 and 10 into Eqn 4, we have the stresses

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix}=\begin{bmatrix}\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16}\\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26}\\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66}\end{bmatrix}\begin{bmatrix}\mathbf{I} & z\mathbf{I}\end{bmatrix}\left(\begin{bmatrix}\overline{\varepsilon}_x\\ \overline{\varepsilon}_y\\ \overline{\gamma}_{xy}\\ \overline{\kappa}_x\\ \overline{\kappa}_y\\ \overline{\kappa}_{xy}\end{bmatrix}-\begin{bmatrix}\begin{bmatrix}1\\1\\0\end{bmatrix}\Delta\Theta_0\\ \begin{bmatrix}1\\1\\0\end{bmatrix}\Delta\Theta_1\end{bmatrix}-\begin{bmatrix}\begin{bmatrix}\cos 2\theta\\ -\cos 2\theta\\ 2\sin 2\theta\end{bmatrix}\Delta\Theta_0'\\ \begin{bmatrix}\cos 2\theta\\ -\cos 2\theta\\ 2\sin 2\theta\end{bmatrix}\Delta\Theta_1'\end{bmatrix}\right)$$

$$=\begin{bmatrix}\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16}\\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26}\\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66}\end{bmatrix}\left(\begin{bmatrix}\mathbf{I} & z\mathbf{I}\end{bmatrix}\left\{\begin{bmatrix}\overline{\varepsilon}_x\\ \overline{\varepsilon}_y\\ \overline{\gamma}_{xy}\\ \overline{\kappa}_x\\ \overline{\kappa}_y\\ \overline{\kappa}_{xy}\end{bmatrix}-\begin{bmatrix}\begin{bmatrix}1\\1\\0\end{bmatrix}\Delta\Theta_0\\ \begin{bmatrix}1\\1\\0\end{bmatrix}\Delta\Theta_1\end{bmatrix}\right\}-\begin{bmatrix}\cos 2\theta & 0 & 0\\ 0 & -\cos 2\theta & 0\\ 0 & 0 & 2\sin 2\theta\end{bmatrix}\begin{bmatrix}\mathbf{I} & z\mathbf{I}\end{bmatrix}\begin{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}\Delta\Theta_0'\\ \begin{bmatrix}1\\1\\1\end{bmatrix}\Delta\Theta_1'\end{bmatrix}\right) \tag{11}$$

Substitution of Eqn 11 into Eqn 5 yields the constitutive equation for the laminate as

$$\begin{bmatrix}N_x\\ N_y\\ N_{xy}\\ M_x\\ M_y\\ M_{xy}\end{bmatrix}=\begin{bmatrix}A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16}\\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26}\\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66}\\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16}\\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26}\\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66}\end{bmatrix}\left(\begin{bmatrix}\overline{\varepsilon}_x\\ \overline{\varepsilon}_y\\ \overline{\gamma}_{xy}\\ \overline{\kappa}_x\\ \overline{\kappa}_y\\ \overline{\kappa}_{xy}\end{bmatrix}-\begin{bmatrix}\overline{\varepsilon}_{x0}\\ \overline{\varepsilon}_{y0}\\ \overline{\gamma}_{xy0}\\ \overline{\kappa}_{x0}\\ \overline{\kappa}_{y0}\\ \overline{\kappa}_{xy0}\end{bmatrix}\right) \tag{12}$$

where

$$A_{ij}=\int\overline{Q}_{ij}dz \qquad B_{ij}=\int\overline{Q}_{ij}zdz \qquad D_{ij}=\int\overline{Q}_{ij}z^2dz \tag{13}$$

and

$$\begin{bmatrix}\overline{\varepsilon}_{x0}\\ \overline{\varepsilon}_{y0}\\ \overline{\gamma}_{xy0}\\ \overline{\kappa}_{x0}\\ \overline{\kappa}_{y0}\\ \overline{\kappa}_{xy0}\end{bmatrix}=\begin{bmatrix}\begin{bmatrix}1\\1\\0\end{bmatrix}\Delta\Theta_0\\ \begin{bmatrix}1\\1\\0\end{bmatrix}\Delta\Theta_1\end{bmatrix}$$

$$+\begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix}^{-1} \begin{bmatrix} A'_{11} & A'_{12} & A'_{16} & B'_{11} & B'_{12} & B'_{16} \\ A'_{12} & A'_{22} & A'_{26} & B'_{12} & B'_{22} & B'_{26} \\ A'_{16} & A'_{26} & A'_{66} & B'_{16} & B'_{26} & B'_{66} \\ B'_{11} & B'_{12} & B'_{16} & D'_{11} & D'_{12} & D'_{16} \\ B'_{12} & B'_{22} & B'_{26} & D'_{12} & D'_{22} & D'_{26} \\ B'_{16} & B'_{26} & B'_{66} & D'_{16} & D'_{26} & D'_{66} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Delta\Theta'_0 \\ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Delta\Theta'_1 \end{bmatrix} \tag{14}$$

in which

$$\begin{bmatrix} A'_{11} & A'_{12} & A'_{16} & B'_{11} & B'_{12} & B'_{16} \\ A'_{12} & A'_{22} & A'_{26} & B'_{12} & B'_{22} & B'_{26} \\ A'_{16} & A'_{26} & A'_{66} & B'_{16} & B'_{26} & B'_{66} \\ B'_{11} & B'_{12} & B'_{16} & D'_{11} & D'_{12} & D'_{16} \\ B'_{12} & B'_{22} & B'_{26} & D'_{12} & D'_{22} & D'_{26} \\ B'_{16} & B'_{26} & B'_{66} & D'_{16} & D'_{26} & D'_{66} \end{bmatrix} = \int \begin{bmatrix} \mathbf{I} \\ z\mathbf{I} \end{bmatrix} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{bmatrix} \cos 2\theta & 0 & 0 \\ 0 & -\cos 2\theta & 0 \\ 0 & 0 & 2\sin 2\theta \end{bmatrix} \begin{bmatrix} \mathbf{I} & z\mathbf{I} \end{bmatrix} dz \tag{15}$$

From Eqn 11, 12 and 14, the stresses can be expressed inters of the generalized stresses and the temperature and moisture changes as

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \left(\begin{bmatrix} \mathbf{I} & z\mathbf{I} \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix}^{-1} \begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} \right.$$

$$+\left\{ \begin{bmatrix} \mathbf{I} & z\mathbf{I} \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix}^{-1} \begin{bmatrix} A'_{11} & A'_{12} & A'_{16} & B'_{11} & B'_{12} & B'_{16} \\ A'_{12} & A'_{22} & A'_{26} & B'_{12} & B'_{22} & B'_{26} \\ A'_{16} & A'_{26} & A'_{66} & B'_{16} & B'_{26} & B'_{66} \\ B'_{11} & B'_{12} & B'_{16} & D'_{11} & D'_{12} & D'_{16} \\ B'_{12} & B'_{22} & B'_{26} & D'_{12} & D'_{22} & D'_{26} \\ B'_{16} & B'_{26} & B'_{66} & D'_{16} & D'_{26} & D'_{66} \end{bmatrix} \right.$$

$$\left. \left. - \begin{bmatrix} \cos 2\theta & 0 & 0 \\ 0 & -\cos 2\theta & 0 \\ 0 & 0 & 2\sin 2\theta \end{bmatrix}_k \begin{bmatrix} \mathbf{I} & z\mathbf{I} \end{bmatrix} \right\} \left[\begin{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Delta\Theta'_0 \\ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Delta\Theta'_1 \end{bmatrix} \right] \right) \tag{16}$$

we can see that the stresses due to hygrothermal loading are expresses in terms of only the deviatoric initial strains DQ_0 and DQ_1.

Superposition Method

We analyze the interlaminar stresses in the vicinity of the end of the terminating dropped ply subjected to the mechanical and hygrothermal loadings by the superposition method. First, we obtain a uniform stress distribution in an infinite laminate without ply drop subjected to the prescribed mechanical and hygrothermal loadings by the classical lamination theory. And we fictitiously remove the plies corresponding to the ply drop as shown in Fig.2(b). We can obtain the tractions acting on the end surfaces of the fictitious cut. Then we apply tractions which are of the same magnitude but of the opposite sign to those exerted on the fictitious cut to the end surfaces of terminating plies in a tapered laminate without any other loadings as shown in Fig.2(c). Superposition of the load system of Fig.2(b) to that of Fig.2(c) gives a laminate with free dropped end surface subjected to the prescribed loadings as shown in Fig.2(a) which reprents the original tapered section problem. Since the load system of Fig.2(b) develops no interlaminar stresses, the transformed problem of Fig.2(c) yields the interlaminar stresses identical to those in the original problem of Fig.2(a). It should be noted that, even if there exist the delaminations in the infinite laminate, the stresses are given by the classical lamination theory. Therefore, a tapered laminate with delaminations at the ends of terminating plies can also be analyzed by the aforementioned superposition method.

In the transformed tapered section problem where the uniform stresses are eliminated and the hygrothermal loading is transformed to the mechanical loading, we can clearly understand the effect of abrupt thickness change and the hygrothermal loading on the interlaminar stress distribution.

POTENTIAL ENERGY RELEASE RATE

For the transformed tapered section problem, the potential energy P is expressed in terms of the strain energy U and the potential of external forces W as

$$\Pi = U + W = -U \tag{17}$$

where the Clapeyron theorem has been utilized. Therefore, the potential energy release rate G associated with the delamination surface increase dA is written as

$$G = -\frac{d\Pi}{dA} = \frac{dU}{dA} \tag{18}$$

For the transformed problem with delaminations at the tapered ends as shown in Fig.3, we can obtain the generalized stresses in the delaminated regions I_t, I_b and II from the tractions applied on the ends of terminating plies and the self-equilibrium conditions of the load system. Taking account of the constraint from the undelaminated region, we assume that

$$\bar{\varepsilon}_y = \bar{\kappa}_y = \bar{\kappa}_{xy} = 0 \tag{19}$$

Then, it follows from Eqn 12 without the initial strains that the generalized stresses contributing to the

strain energy are written as

$$\begin{bmatrix} N_x \\ N_{xy} \\ M_x \end{bmatrix}_i = \begin{bmatrix} A_{11} & A_{16} & B_{11} \\ A_{16} & A_{66} & B_{16} \\ B_{11} & B_{16} & D_{11} \end{bmatrix}_i \begin{bmatrix} \bar{\varepsilon}_x \\ \bar{\varepsilon}_y \\ \bar{\kappa}_x \end{bmatrix}_i \qquad (i = \mathrm{I}_t, \mathrm{I}_b, \mathrm{II}) \tag{20}$$

where the generalized stresses in the region II are given by

$$\begin{bmatrix} N_x \\ N_{xy} \\ M_x \end{bmatrix}_{\mathrm{II}} = -\begin{bmatrix} N_x \\ N_{xy} \\ M_x \end{bmatrix}_{\mathrm{I}_t} - \begin{bmatrix} N_x \\ N_{xy} \\ M_x \end{bmatrix}_{\mathrm{I}_b} \tag{21}$$

If we consider delamination growth da in a laminate with unit width as shown in Fig.3, we get the energy release rate from Eqn 18 as

$$G = \frac{dU}{2da} = \frac{1}{2} \sum_{i=\mathrm{I}_t.\mathrm{I}_b.\mathrm{II}} U_i \tag{22}$$

where U_i is the strain energy per unit length in delaminated region such that

$$U_i = \frac{1}{2}\begin{bmatrix} N_x & N_{xy} & M_x \end{bmatrix}_i \begin{bmatrix} \bar{\varepsilon}_x \\ \bar{\varepsilon}_y \\ \bar{\kappa}_x \end{bmatrix}_i \qquad (i = \mathrm{I}_t, \mathrm{I}_b, \mathrm{II}) \tag{23}$$

From Eqn 20, 22 and 23, we obtain the potential energy release rate as

$$G = \frac{1}{2} \sum_{i=\mathrm{I}_t.\mathrm{I}_b.\mathrm{II}} \begin{bmatrix} N_x & N_{xy} & M_x \end{bmatrix}_i \begin{bmatrix} A_{11} & A_{16} & B_{11} \\ A_{16} & A_{66} & B_{16} \\ B_{11} & B_{16} & D_{11} \end{bmatrix}_i^{-1} \begin{bmatrix} N_x \\ N_{xy} \\ M_x \end{bmatrix}_i \tag{24}$$

RESULTS AND DISCUSSIONS

We obtain the potential energy release rates for the delamination in graphite-epoxy $[q/\text{-}q]_s$ laminates subjected to the mechanical loading $N_x = \bar{N}_x$ and $M_x = \bar{M}_x$ and the temperature and moisture content changes $\Delta T = \Delta\bar{T} = \Delta\bar{T}_0 + \Delta\bar{T}_1 z$ and $\Delta M = \Delta\bar{M} = \Delta\bar{M}_0 + \Delta\bar{M}_1 z$ as shown in Fig.4. Since the lamination $[q/\text{-}q]_s$ is symmetric and $N_y = N_{xy} = M_y = M_{xy} = 0$, it follows from Eqn 12 that

$$\begin{bmatrix} \bar{N}_x \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \left(\begin{bmatrix} \bar{\varepsilon}_x \\ \bar{\varepsilon}_y \\ \bar{\gamma}_{xy} \end{bmatrix} - \begin{bmatrix} \bar{\varepsilon}_{x0} \\ \bar{\varepsilon}_{y0} \\ \bar{\gamma}_{xy0} \end{bmatrix} \right) \tag{25}$$

$$\begin{bmatrix} \overline{M}_x \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \left(\begin{bmatrix} \overline{\kappa}_x \\ \overline{\kappa}_y \\ \overline{\kappa}_{xy} \end{bmatrix} - \begin{bmatrix} \overline{\kappa}_{x0} \\ \overline{\kappa}_{y0} \\ \overline{\kappa}_{xy0} \end{bmatrix} \right) \tag{26}$$

which yield

$$\overline{N}_x = \left[A_{11} - \frac{1}{A_{22}A_{66} - A_{26}^2} \begin{bmatrix} A_{12} & A_{16} \end{bmatrix} \begin{bmatrix} A_{66} & -A_{26} \\ -A_{26} & A_{22} \end{bmatrix} \begin{bmatrix} A_{12} \\ A_{16} \end{bmatrix} \right] \left(\overline{\varepsilon}_x - \overline{\varepsilon}_{x0} \right) \tag{27}$$

$$\overline{M}_x = \left[D_{11} - \frac{1}{D_{22}D_{66} - D_{26}^2} \begin{bmatrix} D_{12} & D_{16} \end{bmatrix} \begin{bmatrix} D_{66} & -D_{26} \\ -D_{26} & D_{22} \end{bmatrix} \begin{bmatrix} D_{12} \\ D_{16} \end{bmatrix} \right] \left(\overline{\kappa}_x - \overline{\kappa}_{x0} \right) \tag{28}$$

Fig. 5 shows the contour plats of the potential energy release rates for graphite-epoxy $[45°/-45°]_s$ tapered laminate under the membrane load $\overline{N}_x$ and the temperature and moisture content changes uniform though the thickness $\Delta\overline{T} = \Delta\overline{T}_0$ and $\Delta\overline{M} = \Delta\overline{M}_0$. It can be seen that the contour lines of the energy release rates in the $\overline{e}_x = \overline{e}_{x_0}$ and $\Delta\overline{\Theta}_0$ plane are elliptic.

Fig. 6 shows the contour plots of the potential energy release rate for graphite-epoxy $[45°/-45°]_s$ tapered laminate under the bending moment $\overline{M}_x$ and the temperature and moisture content change varying linearly through the thickness $\Delta\overline{T} = \Delta\overline{T}_1 z$ and $\Delta\overline{M} = \Delta\overline{M}_1 z$. It can be seen that the contour lines of the energy release rate in the $\left(\overline{\kappa}_x - \overline{\kappa}_{x_0}\right)h/2$ and $\Delta\overline{\Theta}_1' h/2$ plane are elliptic.

CONCLUSIONS

The superposition method was applied to analyze the delaminations at the ends of terminating dropped plies in tapered laminates subjected to the mechanical and hygrothermal loadings. In the transformed tapered section problem developed by the superposition method, the potential energy release rates associated with the delamination growth were obtained. It was found that for the tapered laminate under the axial membrane force $\overline{N}_x$ and axial moment $\overline{M}_x$, and the temperature and moisture changes varying linearly through the thickness such as $\Delta\overline{T} = \Delta\overline{T}_0 + \Delta\overline{T}_1 z$ and $\Delta\overline{M} = \Delta\overline{M}_0 + \Delta\overline{M}_1 z$, the potential energy release rates are expresses in terms of $\overline{e}_x = \overline{e}_{x_0}$, $\overline{\kappa}_x - \overline{\kappa}_{x_0}$, $\Delta\overline{\Theta}_0'$ and $\Delta\overline{\Theta}_1'$ which are given by Eqn 8.2, 27 and 29. It should be noted that the isotropic initial strains given by Eqn 8.1 have no effects on the potential energy release rates.

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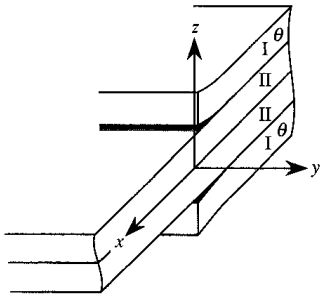

Fig. 1: *Tapered section of laminate under mechanical and hygrothermal loadings.*

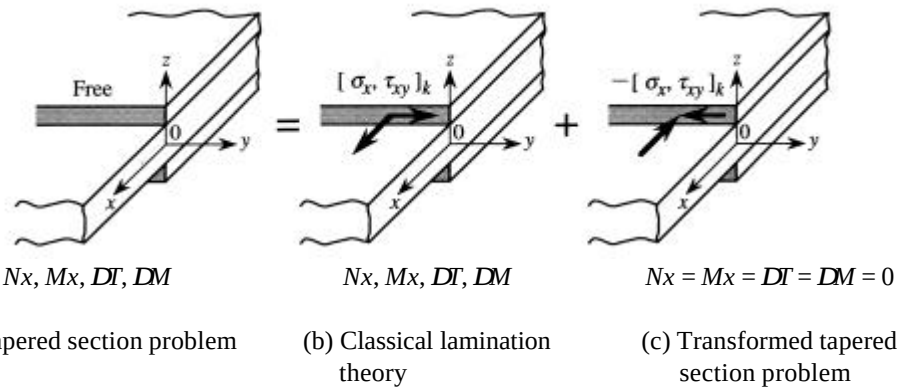

Fig. 2: *Superposition method for analysis of interlaminar stresses in tapered infinite laminate.*

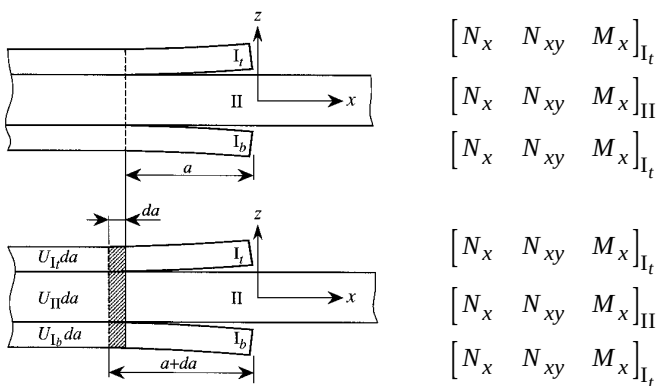

Fig. 3: *Strain energy increase by delamination growth in transformed tapered region problem*

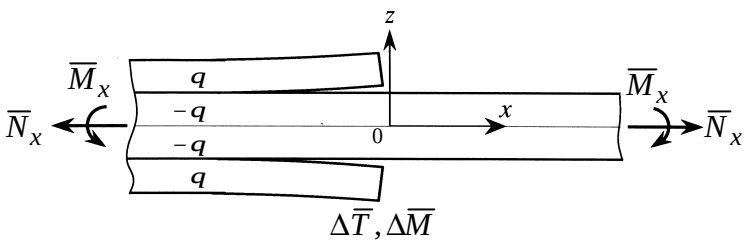

Fig.4: *Graphite-epoxy tapered laminate subjected to mechanical and hygrothermal loadings*

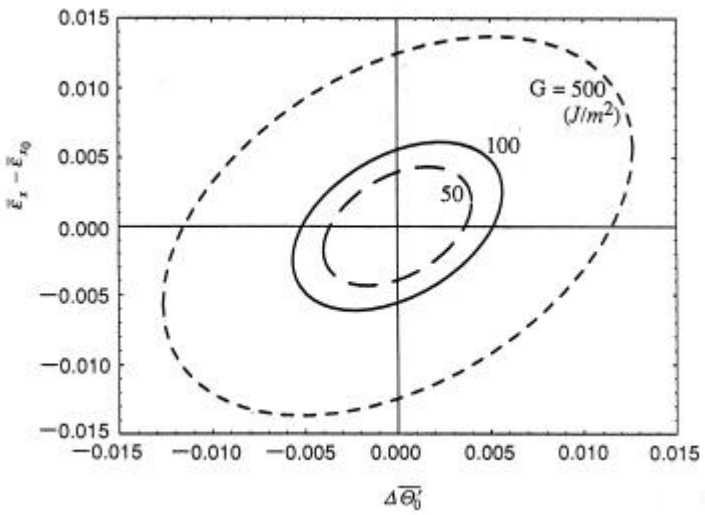

Fig.5 : *Contour plots of potential energy release rates for delamination at tapered section of Graphite-epoxy* $[45°/-45°]_s$ *laminate under membrane load* $\overline{N}_x$ *and temparature and moisture content changes uniform through the thickness* $\Delta\overline{T}_0$ *and* $\Delta\overline{M}_0$.

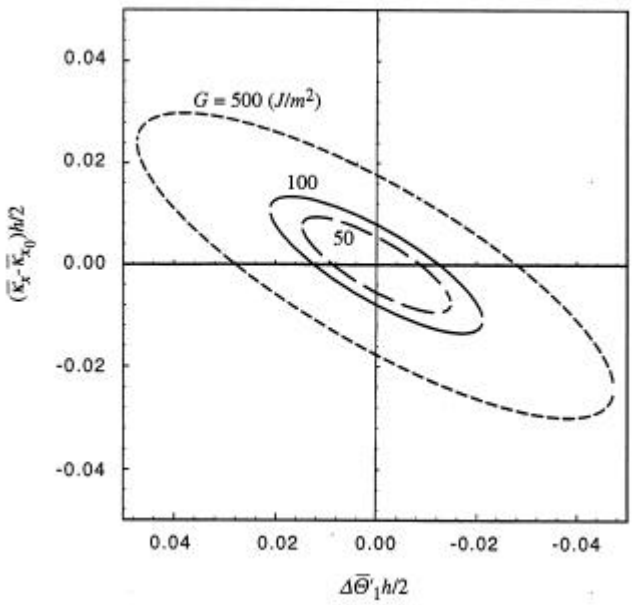

Fig.6 : *Contour plots of potential energy release rates for delamination at tapered section of graphite-epoxy* $[45°/-45°]_s$ *laminate under bending moment* $\overline{M}_x$ *and temperature and moisture content changes varying linearly through the thickness* $\Delta\overline{T}_1 z$ *and* $\Delta\overline{M}_1 z$.