

NUMERICAL SIMULATION OF THE NON LINEAR BEHAVIOUR OF REINFORCED CONCRETE STRUCTURES

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SUMMARY: Many tests carried out on reinforced concrete structures prove that it is not possible to obtain, with a linear calculation, a correct representation of constructions' behaviour. Consequently, developing numerical methods that takes into account different types of non linearity becomes a necessity. The physical one which takes into account the non linearity of concrete behaviour and the geometric one which concerns the overall structure behaviour. The calculation method described here is used to simulate ruin of slender structure made of reinforced concrete and proves that ruin occurs by equilibrium divergence. In this method the structure overall stability is analysed, taking the deflexion into account (second order theory) and considering non linear stress-strain relation of concrete. The comparison of numerical results with experimental results has enabled us to test the used method and to study the impact of various parameters on the critical load computation.

KEYWORDS : Numerical simulation, Slender structures, Reinforced concrete, Non linearity, Buckling.

INTRODUCTION

Reinforced concrete slender structures (hight bridges, piles, cable-stayed bridges pylons) must be calculated using the second order analysis to verify their overall stability[1]. This type of non linear computation is linked to the material nature[2]. We can support this claim by considering an inflected section. It has a tensed part and a compressed part. If the stress exceeds the tensile strength in the tensed part we consider that the concrete is cracked and thus the section inertia reduced[3]. Added to this we consider non linear concrete behaviour with Young's modulus varying with a progressive loading. For statically determinate structures, there are calculation methods using simplifying hypothesis about linear or sinusoidal deflexion to take these phenomena into account[4]. The general aim of this work concerns the development of a numerical method which permits to treat automatically this type of problem for statically determinate and indeterminate structures. A computation program of reinforced concrete section is interfaced with an existing structure application program. It permits to simulate slender structures comportment till the ruin by exceeding the section resistance or by instability. The non linear concrete behaviour is represented by the non linear stress-strain relation parabola-rectangle. This stress-strain relation is used by the French rules of the limit states calculation of reinforced

concrete. The response of the structure is solved in small loading steps by a series of quasi-elastic incremental analysis[5]. The exact solution is the limiting case of infinitely small steps. The study presented in this paper concerns a reinforced concrete statically determinate column loaded by an axial force. This type of structure is rarely perfect therefore an initial eccentricity of the load is taken into account. The numerical simulation results consists to evaluate the critical load of the structure. The results are confronted with experimental results published by the German Committee of Reinforced Concrete[6]. Finally, an evaluation of the parameters' influence on the critical load is done with the geometrical slenderness, the eccentricity of the load, and the concrete compressive strength.

METHOD OF COMPUTATION

Equilibrium method of a reinforced concrete section

The section is subjected to plane bending. Plane sections conservation and small strains hypothesis are used to modelize the section. Concrete behaviour is represented by the stress-strain relation parabola-rectangle as shown in Fig. 1.

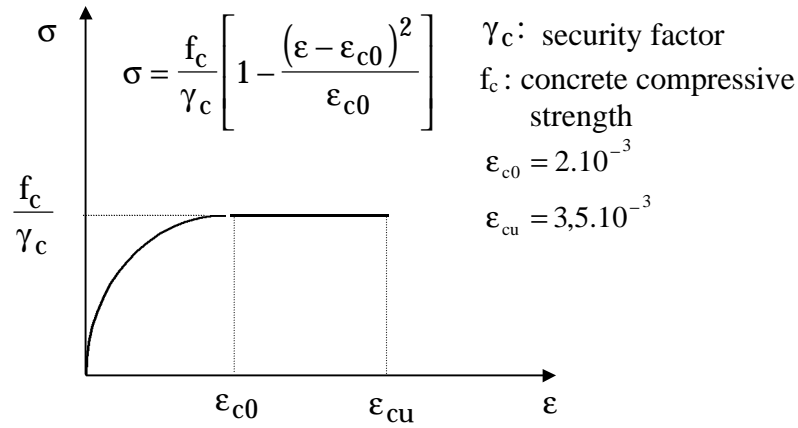


Fig. 1 : Stress-strain relation parabola-rectangle

In this study, the non-linear analysis concerns axial force N_{ext} and bending moment M_{ext} which are applied to xOy plane. The axial strain for a typical point of the cross section is given by Eqn 1:

$$\epsilon = \delta u + \delta \omega \cdot y \quad (1)$$

δu is the average compressive strain, $\delta \omega$ is the differential rotation and y is the ordinate of the typical point of the cross section. The cross-section is a rectangle of concrete which is defined by its larger b and height h and different steel reinforcements which are defined by their ordinate y_i and their section area A_i . Fig. 2 represents a reinforced concrete section subjected to plane bending.

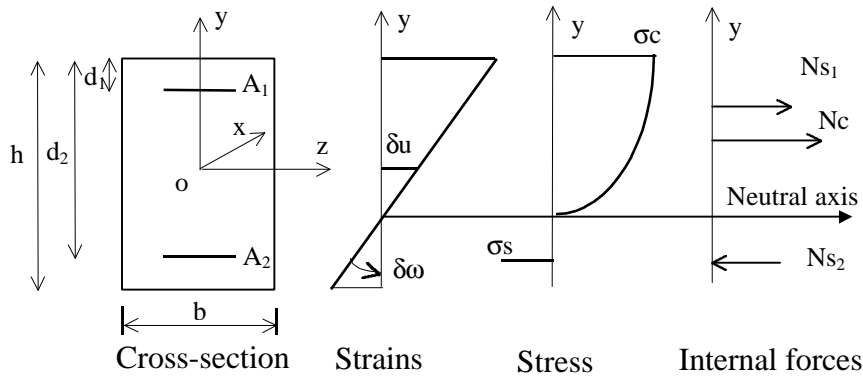


Fig. 2 : reinforced concrete cross-section loaded in plane bending

Internal forces in concrete are given by Eqn 2 and Eqn 3 :

$$N_c = \int_h \sigma_c(\delta u + \delta \omega y) b(y) dy \quad (2)$$

$$M_c = \int_h \sigma_c(\delta u + \delta \omega y) y b(y) dy \quad (3)$$

Internal forces in reinforcement are given by Eqn 4 and Eqn 5:

$$N_s = \sum_{i=1}^n A_i \sigma_s(\delta u + \delta \omega y_i) \quad (4)$$

$$M_s = \sum_{i=1}^n A_i \sigma_s(\delta u + \delta \omega y_i) y_i \quad (5)$$

Total internal forces in reinforced concrete cross-section are given by Eqn 6 and Eqn 7:

$$N_{int} = N_c + N_s \quad (6)$$

$$M_{int} = M_c + M_s \quad (7)$$

Let us consider that Φ is the operator which permits to have forces produced by given strain. This relation is given by the Eqn 8 as follows:

$$\begin{Bmatrix} N_{int} \\ M_{int} \end{Bmatrix}(\delta u, \delta \omega) = \Phi(\delta u, \delta \omega) \quad (8)$$

Due to the behaviour of concrete (parabola-rectangle), the problem is non-linear therefore forces N_c and M_c are non linear functions of $(\delta u, \delta \omega)$. The principle is to approximate calculation by considering a linear problem locally and by proceeding by increase of strains and by estimating tangentially operator Φ_t . For this, we use an iterative method to compute strains. The principle of this method is shown in Fig 3.

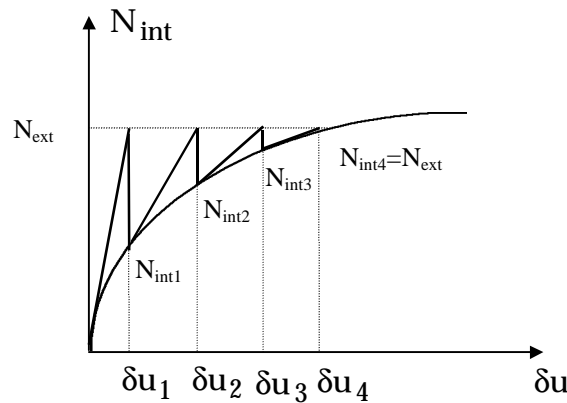


Fig. 3 : Iterative calculation of non linear problem reduced to an only variable δu . Convergence towards an equilibrium state.

For given forces (N_{ext}, M_{ext}) calculation is organised as follows :

-we start from a given strain ($\delta u, \delta \omega$) ; it is generally given by linear elasticity by Eqn 9 and Eqn 10:

$$\delta u = \frac{N_{\text{ext}}}{ES} \quad (9)$$

$$\delta \omega = \frac{M_{\text{ext}}}{EI} \quad (10)$$

- we compute the internal forces ($N_{\text{int}}, M_{\text{int}}$) ;
- we calculate the tangential operator Φ_t ;
- we evaluate the difference between the internal and the external forces ;
- the actual strain is given by Eqn 11:

$$\begin{Bmatrix} \delta u_{i+1} \\ \delta \omega_{i+1} \end{Bmatrix} = \begin{Bmatrix} \delta u_i \\ \delta \omega_i \end{Bmatrix} + [\Phi_t (\delta u_i + \delta \omega_i)]^{-1} \begin{Bmatrix} N_{\text{ext}} - N_{\text{int } i} \\ M_{\text{ext}} - M_{\text{int } i} \end{Bmatrix} \quad (11)$$

Computations are stopped when $(N_{\text{int}}, M_{\text{int}}) = (N_{\text{ext}}, M_{\text{ext}})$.

Column stability

To study a statically determinate column we discretize it in n elements therefore in n cross-sections. The principle is to analyse each cross-section equilibrium and to prove that there is stability of equilibrium. This must be made with the two types of non linearity. The stability analysis is made with an incremental loading. If the shear forces are neglected, the first order forces are axial force N_0 and first order bending moment $(M_x)_0$. Due to column deflection (Fig. 4), the resultant forces takes a new position x_i . This position gives a new stress distribution including the second order effects calculated with Eqn 12 and Eqn 13:

$$N_i = N_0, \quad (M_x)_i = (M_x)_0 + N_0 \cdot x_i \quad (12)$$

$$N_{i+n} = N_0, \quad (M_x)_{i+n} = (M_x)_{i+(n-1)} + N_0 \cdot x_{i+n} \quad (13)$$

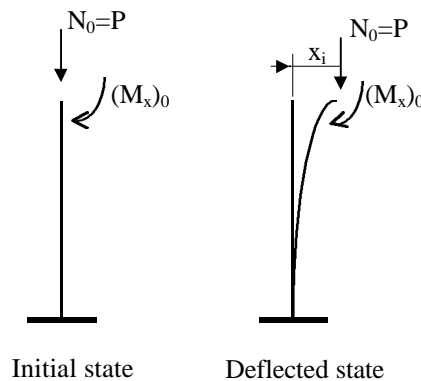


Fig. 4 : Second order effects on reinforced concrete column

An iterative algorithm is established for computing the second order bending moment which grows for the same increment due to deflexion. Using successive iterations, the stability criterion is the convergence of bending moment values towards finite values.

Analysis organisation

For a discretised column the analysis is made with a step by step loading. For every step the stability analysis is organised as follows :

- computation of the first order forces ;
- computation in each cross-section of the strains with the equilibrium method ;
- evaluation of the neutral axis, the inertia (tensed concrete is neglected), and the tangent young's modulus in each cross-section;
- computation of the second order forces and testing of the stability criterion.

This organisation is summarized by an organigram in Fig. 5.

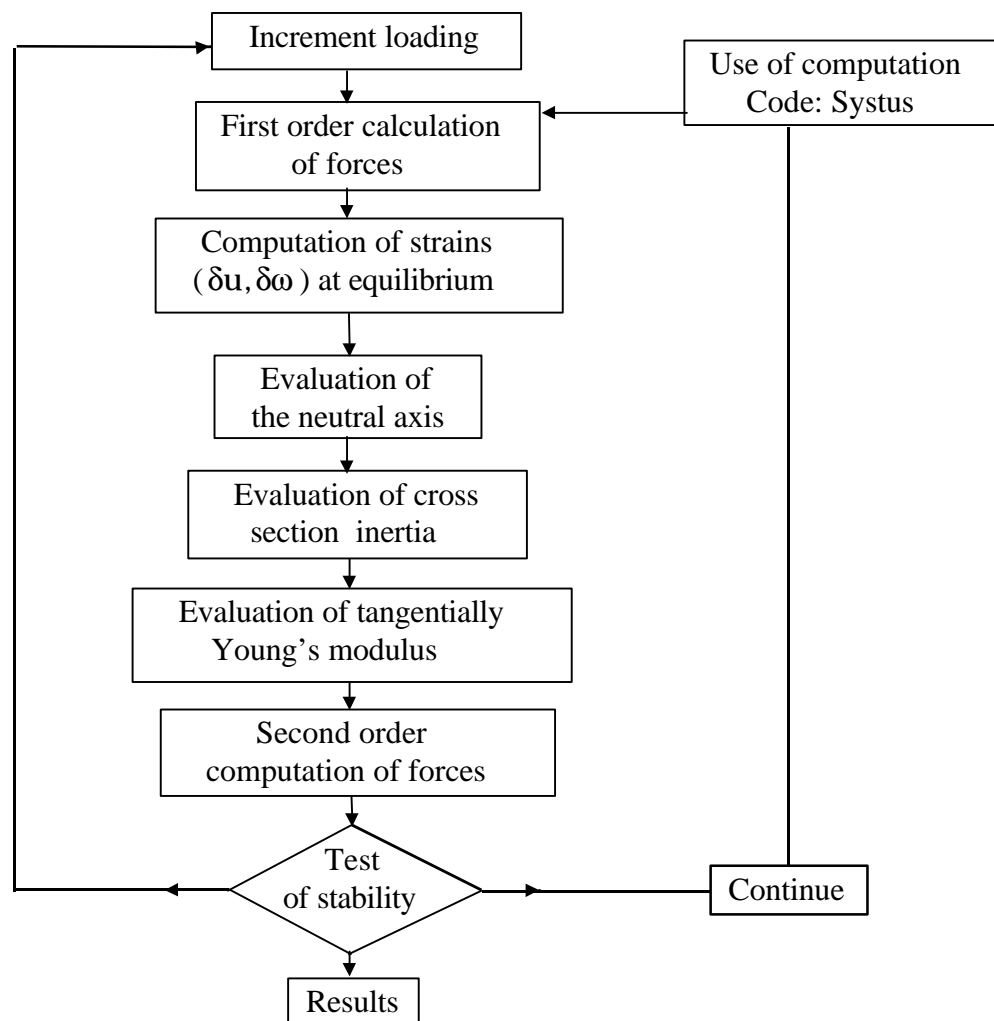


Fig. 5 : Organisation of a step computation

Computation code and programming language

The first order calculations are made by a finite element code named Systus. It is used in various domains and particularly in civil engineering. It gives a vast range of procedures which permits to make

many analyses in static or dynamic problems. The programming language SIL is integrated into the code. It represents an interface language with the database.

VALIDATION OF THE USED METHOD

A series of statically determinate columns have been tested by Mehmél and his collaborators and results have been published by the German Committee of Reinforced Concrete[6]. We have used these tests to valid the used method . The validation concerns columns that have one end restrained and one free end. The column is loaded with an axial load P with an eccentricity e_0 . The columns characteristics are presented in the Table 1 and numerical results comparison with the experimental results is presented in the Table 2. The columns characteristics are :

- the section dimension h in the buckling plane;
- the section dimension b in the orthogonal buckling plane;
- the compressed reinforcement section A_1 ;
- the tensed reinforcement section A_2 ;
- the distance d_1 from the most compressed section fibre to the reinforcement section A_1 ;
- the distance d_2 from the most compressed section fibre to the reinforcement section A_2 ;

Table 1: Columns characteristics

test	l cm	h cm	b cm	$A_1=A_2$ cm	d_1 cm	d_2 cm	f_e MPa	f_c MPa	e_0 cm
1	450	20,3	25,2	3,12	3,2	17,1	480	35,3	9,7
2	450	15	25,3	2,36	2,5	12,3	510	42,3	7,3
3	340	15,6	25,2	2,38	2,6	12,5	510	41,9	7,6

Table 2: Numerical results comparison with the experimental results

test	P_{exp} MN	P_{inf} MN	P_{sup} MN
1	0,260	0,250	0,264
2	0,138	0,120	0,140
3	0,180	0,170	0,190

The numerical calculation gives two values of load. The critical load value is between the last load value for which the analysis converged (P_{inf}) and the load value for which the analysis diverge (P_{sup}). According to Table 2 we can say that we obtain a good estimation of the experimental ruin load.

IMPACT OF VARIOUS PARAMETERS ON THE CRITICAL LOAD

After the method validation it is possible to study the impact of some parameters on the column critical load P_c ; geometrically slenderness, load eccentricity, compressive strength of concrete. The characteristics column are $l = 450$ cm, $h = 15$ cm, $b = 25,3$ cm, $A = A' = 2,36$ cm², $d = 12,3$ cm, $d' = 2,5$ cm.

Influence of geometrical slenderness

The evolution of reduced critical load $\frac{P_c}{Bf_c}$ when the geometrical slenderness $\frac{l_f}{h}$ varies is shown in

Fig. 6 (B is the concrete section area). The column effective length is l_f . Three distinct zones can be observed on the curve. The first one corresponds to small slenderness values for which the critical load does not vary much. In the second one we can see a great decrease in critical load; for standard elements whose slenderness values are between 15 and 25, 10% of slenderness variation entails the same order critical load variation. In the last zone the curve is stabilised for extensive geometrical slenderness. In this case the critical loads are all the smaller as the columns are more slender.

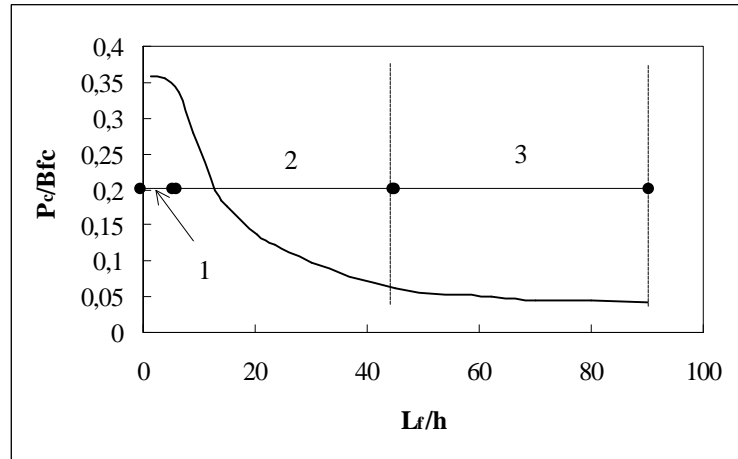


Fig. 6: Critical load versus geometrical slenderness

Influence of load eccentricity

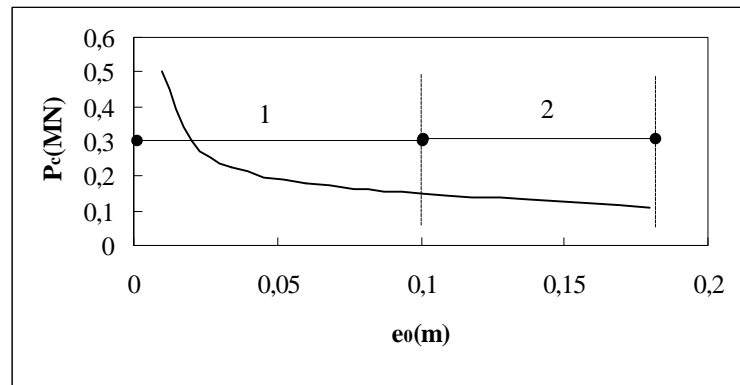


Fig. 7 : critical load versus eccentricity

The critical load P_c versus load eccentricity e_0 is given in Fig. 7. We can observe two zones on the curve. The first zone shows an important decrease of critical load values for small eccentricities; for example between $e_0 = 1$ cm and $e_0 = 2$ cm we can note a 38% diminution in the critical load; this proves that it is dangerous to neglect or underestimate an initial eccentricity due to the structures' imperfections in buckling calculation. The second zone proves that for high eccentricity values the curve is stabilised; for very outlying loads it is logical that the column instability occur immediately.

Influence of concrete compressive strength

The evolution of critical load P_c when compressive strength of concrete varies is shown on Fig. 8. The critical load has a quasi linear increase versus compressive strength of concrete. This linearity can be explained by the fact that the problem is resolved by a linear incremental analysis and that the Young's modulus is a linear function of compressive strength f_c ($E = k f_c$; k is constant).

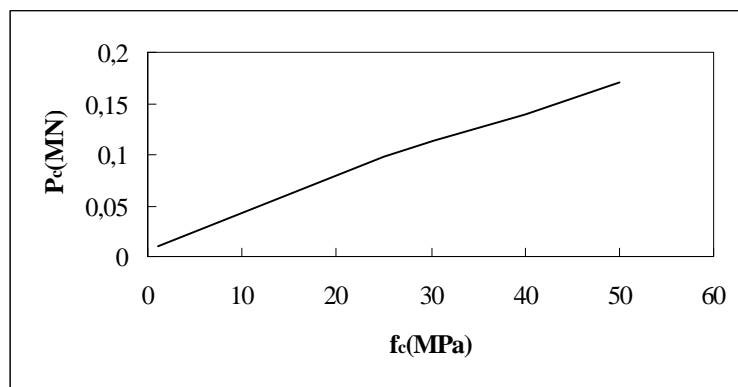


Fig. 8 : critical load versus compressive strength of concrete

Others parameters

Steel yield limit has no influence or almost no influence on the critical load calculation because stability generally occurs when steel yield limit is not reached.

Finally, for two values of reinforcement position in the section $d_1 = 3,5$ cm et $d_1 = 4,5$ cm, we obtain respectively the critical loads $P = 0,16$ MN et $P = 0,14$ MN. There is a 13% variation of the critical load for a mistake of 1 cm on the reinforcement position. Therefore, it could be wise to increase the reinforcement position d_1 and d_2 in critical load calculation.

CONCLUSION

The developed program permits to simulate the behaviour till ruin of reinforced concrete columns loaded by axial force in the short term. Ruin can occur by exceeding the strength of the more solicited section or by instability. This program can be considered like a tool of buckling verification in the case of statically determinate structures. The problem is more complex when it is about statically indeterminate structures because of forces redistribution due to the rigidity variation for incremental loading. We are at present studying this problem. The numerical results will be compared with results obtained on experimental patterns. Moreover using other concrete strain-stress relation in the short term will permit to provide more information about physical non linearity. Finally with the introduction in the program of the concrete and steel long term behaviour, we will evaluate the impact of this phenomenon on the stability of concrete structures.

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