THE OPTIMIZED FIBER VOLUME FRACTION FOR MAGNETOELECTRIC COMPOSITES

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SUMMARY: Explicit formulae are presented to assess the optimized fiber volume fraction for maximum magnetoelectric coupling effect exhibiting in the composite with piezomagnetic matrix reinforced by piezoelectric continuous fibers. The formulae consist of the analytical expressions for the effective properties of the composite and the magneto-electro-elastic Eshelby tensors. With these expressions, the closed-form solution for the magnetoelectric coupling effect, which is a new property existing in the piezoelectric-piezomagnetic composite even though neither of the individual phases exhibits such a property, is acquired. Furthermore, by taking the derivative of the close form for magnetoelectric coupling effect with respect to the fiber volume fraction as zero yields the optimized volume fraction of fibers analytically. The optimized fiber volume fraction shows that it is a function of the elastic properties of constituents, but not a function of the magnetic and electric properties.

KEYWORDS: piezoelectric/piezomagnetic composites, the magneto-electro-elastic Eshelby tensors, magnetoelectric coupling effect.

INTRODUCTION

Because of a unique property in terms of magnetoelectric coupling effect resulting from the interaction between magnetism, electricity, and mechanism, piezoelectric/piezomagnetic composites attract increasing attention for the design of intelligent or active structures. They are typically used to generate acoustic pulse from an input mechanical loading (or magnetic signal, or electric signal) and to detect an acoustic pulse and then convert it into an electric (or magnetic) signal. A remarkably wide variety of applications include the development of electronic packaging, magneto-electro-mechanical transducers, magnetic sensors, strain gages, infrared absorbers, chemical sensors, and acoustic actuators. Particularly, their applications in magnetic sensors could be very helpful for detecting both *ac* and *dc* magnetic fields. This suggests potential usage in magnetic storage and read-out devices, in magnetic imaging technology, and for protecting database by sensing and shielding from detrimental magnetic fields [1].

Piezoelectric/piezomagnetic composites can be fabricated in the form of secondary-phase

piezoelectric inclusions embedded in a piezomagnetic matrix, or vice versa. It is recognized that the inclusion volume fraction has significant influences not only on mechanical properties but also on magnetoelectric coupling effect of piezoelectric/piezomagnetic composites. However, the change in the composite's properties with compositional variations cannot usually be described using a simple formalism; thus, the fundamental study of the properties of the composite materials remains an active area of basic and applied research. Avellaneda and Harshe [1], Harshe [2], Harshe and co-workers [3], Nan [4], and Benveniste [5] have investigated the magnetoelectric coupling effect in piezoelectric/piezomagnetic composite materials. However, their solutions for magnetoelectric coupling effect are not explicit. Recently, Huang and Kuo [6] have derived the expression for the coupled elastic, electric, and magnetic fields in the unified form of surface integral and the evaluation of the fields can be followed by numerical computation. Nevertheless, if a closed form solution can be obtained, it can apply extensively to piezoelectric/piezomagnetic composites with various inclusion volume fraction, inclusion shape, and material systems without tedious calculation.
Therefore, this paper attempts to explore the magneto-electro-elastic Eshelby tensors defined by Eqn 20 in previous work [6] explicitly for continuous fibers in a transversely isotropic piezoelectric/piezomagnetic composite. Consisting of piezoelectric continuous fibers in a piezomagnetic matrix, the composite has highest magnetoelectric coupling effect among composites with various inclusion shapes. Once the explicit expression is obtained, the composition and inclusion shape-dependent complex magnetoelectric coupling effect can be examined. Finally, optimized inclusion volume fractions can be obtained for various material systems with maximum magnetoelectric coupling effect.

SOME PRELIMINARIES

The Inclusion Problems

Before proceeding some notations used in this article are introduced. The usual summation convention applies to repeated subscripts with the exception that both lowercase and uppercase subscripts are used. Lowercase subscripts take on the range *1, 2, 3,* while uppercase subscripts range from *1* to *5*. Thus, $T_J U_J = T_j U_j + T_4 U_4 + T_5 U_5$, where $j = 1 \rightarrow 3$. With this shorthand notation, the magneto-electro-elastic moduli of a piezomagnetic material is conveniently expressed as

$$L^0_{iJMn} = \begin{cases} C^0_{ijmn} & J, M \le 3, \\ q^0_{nij} & J \le 3, \ \ M = 5, \\ q^0_{imn} & J = 5, \ \ M \le 3, \\ -\kappa^0_{in} & J = M = 4, \\ -\Gamma^0_{in} & J = M = 5, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Similarly the magneto-electro-elastic moduli of a piezoelectric material is represented as

$$L^1_{iJMn} = \begin{cases} C^1_{ijmn} & J, M \le 3, \\ e^1_{nij} & J \le 3, \quad M = 4, \\ e^1_{imn} & J = 4, \quad M \le 3, \\ -\kappa^1_{in} & J = 4, \quad M = 4, \\ -\Gamma^1_{in} & J = 5, \quad M = 5, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

In the preceding equations, the superscripts '0' and '1' denote quantities in the matrix and the inhomogeneity respectively; C_{ijmn} denotes elastic moduli, e_{nij} piezoelectric coefficient, q_{nij} piezomagnetic coefficient, κ_{in} dielectric constant, and Γ_{in} magnetic permeability. It is noted that, in general, the dielectric constant and magnetic permeability are neglected for piezoelectric and piezomagnetic materials, respectively. However, they are retained in the present work in order to investigate the magnetoelectric coupling effect in piezoelectric-piezomagnetic composites.

Now, consider an infinite piezomagnetic material containing an ellipsoidal inclusion whose material properties are the same as the matrix and is defined as

$$, \ \Omega : \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2} + \frac{x_3^2}{a_3^2} \le 1 \tag{3}$$

where a_1, a_2 , and a_3 are the lengths of the semiaxes of the ellipsoid. Let Z^*_{Mn} represent eigenstrain [7] (or stress-free transformation strain ε^*_{mn} when $M \le 3$), eigenelectric field (or electric displacement-free electric field $-E^*_n$ when $M = 4$), and eigenmagnetic field (or magnetic induction-free magnetic field $-H^*_n$ when $M = 5$) in the inclusion, and zero in the matrix. The stress σ_{ij}, the electric displacement D_i, and the magnetic induction B_i in the inclusion caused by a constant Z^*_{Mn} in Ω can be expressed as:

$$\begin{aligned} \sigma_{ij} &= C^0_{ijmn}(\varepsilon_{mn} - \varepsilon^*_{mn}) - q^0_{nij}(H_n - H^*_n), \\ D_i &= \kappa^0_{in}(E_n - E^*_n), \\ B_i &= q^0_{imn}(\varepsilon_{mn} - \varepsilon^*_{mn}) + \Gamma^0_{in}(H_n - H^*_n), \end{aligned} \tag{4}$$

in which ε_{mn} represents elastic strain tensor, E_n electric field, and H_n magnetic field. In the shorthand notation, the constitutive Eqn 4 can be unifined into a single equation:

$$\Sigma_{iJ} = L^0_{iJMn}(Z_{Mn} - Z^*_{Mn}), \tag{5}$$

where

$$\Sigma_{iJ} = \begin{cases} \sigma_{ij} & J \le 3, \\ D_i & J = 4, \\ B_i & J = 5, \end{cases} \qquad Z_{Mn} = \begin{cases} \varepsilon_{mn} = S_{mnAb} Z^*_{Ab} & M \le 3, \\ -E_n = S_{4nAb} Z^*_{Ab} & M = 4, \\ -H_n = S_{5nAb} Z^*_{Ab} & M = 5, \end{cases} \tag{6}$$

with S_{MnAb} being a collection of 9 tensors that are referred to as the magneto-electro-elastic Eshelby tensors analogous to Eshelby [8] tensor for elastic inclusion problems. As will be seen in the subsequent development, the magneto-electro-elastic Eshelby tensors are the key ingredients necessary for determining the magnetoelectric coupling of piezoelectric-piezomagnetic composites. It is useful to express S_{MnAb} explicitly in terms of the magneto-electro-elastic moduli L^0_{iJMn}, i.e.,

$$
\begin{aligned}
&S_{mnab} = \frac{1}{8\pi}\left\{C^0_{ijab}(G_{mjin} + G_{njim}) + q^0_{iab}(G_{m5in} + G_{n5im})\right\},\\
&S_{mn4b} = \frac{-1}{8\pi}\kappa^0_{ib}(G_{m4in} + G_{n4im}),\\
&S_{mn5b} = \frac{1}{8\pi}\left\{q^0_{bij}(G_{mjin} + G_{njim}) - \Gamma^0_{ib}(G_{m5in} + G_{n5im})\right\},\\
&S_{4nab} = \frac{1}{4\pi}(C^0_{ijab}G_{4jin} + q^0_{iab}G_{45in}), \qquad S_{4n4b} = -\frac{1}{4\pi}\kappa^0_{ib}G_{44in}\\
&S_{4n5b} = \frac{1}{4\pi}(q^0_{bij}G_{4jin} - \Gamma^0_{ib}G_{45in}), \qquad S_{5nab} = \frac{1}{4\pi}(C^0_{ijab}G_{5jin} + q^0_{iab}G_{55in}),\\
&S_{5n4b} = -\frac{1}{4\pi}\kappa^0_{ib}G_{54in}, \qquad S_{5n5b} = \frac{1}{4\pi}(q^0_{bij}G_{5jin} - \Gamma^0_{ib}G_{55in}),
\end{aligned}
\tag{7}
$$

in which G_{MJin} is defined by [6]

$$
G_{MJin} = \int_{-1}^{1}\int_{0}^{2\pi} N_{MJ}(\overline{\mathbf{\xi i}})D^{-1}(\overline{\mathbf{\xi i}})\overline{\xi}_i\overline{\xi}_n d\theta d\overline{\varsigma}_3, \tag{8}
$$

with $N_{MJ}(\overline{\mathbf{\xi i}})$ and $D(\overline{\mathbf{\xi i}})$ being the cofactor and the determinant of the 5×5 matrix $[L^0_{iJMn}\overline{\xi}_i\overline{\xi}_n]$.

Effective Magneto-Electro-Elastic Moduli

Suppose that a sufficiently large two-phase composite consists of randomly distributed piezoelectric ellipsoidal inhomogeneities ($\Omega_1, \Omega_2, \cdots, \Omega_N$) with magneto-electro-elastic moduli L^1_{iJMn} and volume fraction *f*. The surrounding matrix is piezomagntic and has magneto-electro-elastic moduli L^0_{iJMn}. The effective magneto-electro-elastic moduli $\overline{L}_{iJMn}$ of the composite have been obtained by Huang and Kuo [6] through the Mori-Tanaka [9] theory incorporated with the equivalent inclusion method as

$$
\overline{L}_{iJMn} = L^0_{iJAb} + fL^0_{iJAb}V^{-1}_{AbqR}(L^1_{qRMn} - L^0_{qRMn} \quad , \tag{9}
$$

where V^{-1}_{AbiJ} is the inverse of V_{iJAb} defined by

$$
V_{iJAb} = (1-f)(L^1_{iJMn} - L^0_{iJMn})S_{MnAb} + L^0_{iJAb}. \tag{10}
$$

Due to the coupling interaction between magnetostriction of piezomagnetic phase and piezoelectricity of piezoelectric phase, the effective composite properties $\overline{L}_{iJMn}$ should be comprised of the magnetoelectric coupling effect. Thus, the unified notation $\overline{L}_{iJMn}$ in Eqn 9 is defined as:

$$\overline{L}_{iJMn} = \begin{cases} \overline{C}_{ijmn} & J, M \le 3, \\ \overline{e}_{nij} & J \le 3, \ \ M = 4, \\ \overline{q}_{nij} & J \le 3, \ \ M = 5, \\ \overline{e}_{imn} & J = 4, \ \ M \le 3, \\ -\overline{\kappa}_{in} & J = 4, \ \ M = 4, \\ -\overline{\lambda}_{in} & J = 4, \ \ M = 5; J = 5, \quad M = 4, \\ \overline{q}_{imn} & J = 5, \ \ M \le 3, \\ -\overline{\Gamma}_{in} & J = 5, \ \ M = 5, \end{cases} \tag{11}$$

in which $\overline{\lambda}_{in}$ stands for the magnetoelectric coupling factor that is absent in the constituents of the composite. Thus, the overall stress, electric displacement, and magnetic induction in the composite are expressed as

$$\begin{aligned} \overline{\sigma}_{ij} &= \overline{C}_{ijmn}\overline{\varepsilon}_{mn} - \overline{e}_{nij}\overline{E}_n - \overline{q}_{nij}\overline{H}_n, \\ \overline{D}_i &= \overline{e}_{imn}\overline{\varepsilon}_{mn} + \overline{\kappa}_{in}\overline{E}_n + \overline{\lambda}_{ni}\overline{H}_n, \\ \overline{B}_i &= \overline{q}_{imn}\overline{\varepsilon}_{mn} + \overline{\lambda}_{in}\overline{E}_n + \overline{\Gamma}_{in}\overline{H}_n, \end{aligned} \tag{12}$$

or in the unified notation

$$\overline{\Sigma}_{iJ} = \overline{L}_{iJMn}\overline{Z}_{Mn}, \tag{13}$$

where the overbar denotes quantities associated with the entire composite.

MAGNETO-ELECTRO-ELASTIC ESHELBY TENSORS FOR CONTINUOUS FIBERS

Next, we attempt to explore the magneto-electro-elastic Eshelby tensors analytically for a practical inclusion in the micromechanics and composite communities: piezoelectric continuous fibers (circular cylindrical inclusions: $a_1 = a_2$, $a_3 \to \infty$) embedded in transversely isotropic piezomagnetic materials. To this end, completely explicit expressions of the determinant $D(\overline{\xi})$ and the non-zero components of the cofactor $N_{MJ}(\overline{\xi})$ are carried out as

$$D(\overline{\xi}) = \frac{1}{a_1^{10}} b_{11}(\eta_1^{10} + \eta_2^{10} + \eta_1^8\eta_2^2 + \eta_1^2\eta_2^8 + 2\eta_1^6\eta_2^4 + 2\eta_1^4\eta_2^6),$$

$$N_{11}(\overline{\xi}) = N_{22}(\overline{\xi}) = \frac{1}{a_1^8} b_{11}(\eta_1^8 + \eta_2^8 + \eta_1^6\eta_2^2 + \eta_1^2\eta_2^6 + \eta_1^4\eta_2^4),$$

$$N_{33}(\overline{\xi}) = \frac{1}{a_1^8}\left\{b_{33}(\eta_1^8 + \eta_2^8) + g_{33}(\eta_1^6\eta_2^2 + \eta_1^2\eta_2^6) + i_{33}\eta_1^4\eta_2^4\right\},$$

$$N_{35}(\bar{\xi}) = N_{53}(\bar{\xi}) = \frac{1}{a_1^8}\left\{b_{35}(\eta_1^8 + \eta_2^8) + g_{35}(\eta_1^6\eta_2^2 + \eta_1^2\eta_2^6) + i_{35}\eta_1^4\eta_2^4\right\},$$

$$N_{44}(\bar{\xi}) = \frac{1}{a_1^8}\left\{b_{44}(\eta_1^8 + \eta_2^8) + g_{44}(\eta_1^6\eta_2^2 + \eta_1^2\eta_2^6) + i_{44}\eta_1^4\eta_2^4\right\},$$

$$N_{55}(\bar{\xi}) = \frac{1}{a_1^8}\left\{b_{55}(\eta_1^8 + \eta_2^8) + g_{55}(\eta_1^6\eta_2^2 + \eta_1^2\eta_2^6) + i_{55}\eta_1^4\eta_2^4\right\},$$

$$N_{12}(\bar{\xi}) = N_{21}(\bar{\xi}) = \frac{\eta_1\eta_2}{a_1^8}\left\{b_{12}(\eta_1^4\eta_2^2 + \eta_1^2\eta_2^4) + g_{12}(\eta_2^6 + \eta_1^6)\right\}, \tag{14}$$

where $\eta_1 = \sqrt{1-\eta_3}\cos\theta$, $\eta_2 = \sqrt{1-\eta_3}\sin\theta$, and $b_{11} \sim i_{55}$ are listed in Appendix A. Upon substituting Eqn 14 into Eqn 8 and evaluating the double integrals in G_{MJin}, the nonzero components of the Eshelby tensors S_{MnAb} in Eqn 7 can then be obtained in the following closed forms.

$$S_{1111} = S_{2222} = \frac{5\,C_{11}^0 + C_{12}^0}{8C_{11}^0},\ S_{1122} = S_{2211} = \frac{-C_{11}^0 + 3C_{12}^0}{8C_{11}^0},$$

$$S_{1133} = S_{2233} = \frac{C_{13}^0}{2C_{11}^0}, \qquad S_{1153} = S_{2253} = \frac{q_{31}^0}{2C_{11}^0},$$

$$S_{1212} = S_{1221} = S_{2112} = S_{2121} = \frac{3C_{11}^0 - C_{12}^0}{8C_{11}^0},$$

$$S_{1313} = S_{1331} = S_{3113} = S_{3131} = S_{2323} = S_{2332} = S_{3223} = S_{3232} = \frac{1}{4},$$

$$S_{4141} = S_{4242} = S_{5151} = S_{5252} = \frac{1}{2}. \tag{15}$$

THE OPTIMIZED FIBER VOLUME FRACTION FOR MAGNETOELECTRIC COUPLING EFFECT

With the explicit model for the effective magneto-electro-elastic moduli proposed in section 2 and the analytical expressions for the magneto-electro-elastic Eshelby tensors in section 3, the magnetoelectric coupling effect of piezoelectric-piezomagnetic composites will be investigated analytically in this section. It is readily shown from Eqn 9 with the aid of Eqn 11 that

$$\bar{\lambda}_{11} = -f\{2\kappa_{11}^0 q_{15}^0 V_{4113}^{-1} + (\Gamma_{11}^1 - \Gamma_{11}^0)V_{4115}^{-1}\},$$
$$\bar{\lambda}_{22} = -f\{2\kappa_{11}^0 q_{15}^0 V_{4223}^{-1} + (\Gamma_{11}^1 - \Gamma_{11}^0)V_{4225}^{-1}\},$$
$$\bar{\lambda}_{33} = -f\{\kappa_{33}^0 q_{31}^0 (V_{4311}^{-1} + V_{4322}^{-1}) + \kappa_{33}^0 q_{33}^0 V_{4333}^{-1} + (\Gamma_{33}^1 - \Gamma_{33}^0)V_{4335}^{-1}\}, \tag{16}$$

and $\bar{\lambda}_{ij} = 0$ otherwise. By inspection of the equations above it is seen that to evaluate the magnetoelectric coupling effect analytically, the inversion of the fourth-order tensor V_{iJAb} has to be carried out before proceeding any further. For an ellipsoidal inclusion embedded in transversely isotropic piezomagnetic media, the inversion of V_{iJAb} are given in the following matrix:

$$
\begin{bmatrix}
V_{1111}^{-1} & V_{1122}^{-1} & V_{1133}^{-1} & 0 & 0 & 0 & 0 & 0 & V_{1134}^{-1} & 0 & 0 & V_{1135}^{-1} \\
V_{2211}^{-1} & V_{2222}^{-1} & V_{2233}^{-1} & 0 & 0 & 0 & 0 & 0 & V_{2234}^{-1} & 0 & 0 & V_{2235}^{-1} \\
V_{3311}^{-1} & V_{3322}^{-1} & V_{3333}^{-1} & 0 & 0 & 0 & 0 & 0 & V_{3334}^{-1} & 0 & 0 & V_{3335}^{-1} \\
0 & 0 & 0 & V_{2323}^{-1} & 0 & 0 & 0 & V_{2324}^{-1} & 0 & 0 & V_{2325}^{-1} & 0 \\
0 & 0 & 0 & 0 & V_{1313}^{-1} & 0 & V_{1314}^{-1} & 0 & 0 & V_{1315}^{-1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & V_{1212}^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & V_{4113}^{-1} & 0 & V_{4114}^{-1} & 0 & 0 & V_{4115}^{-1} & 0 & 0 \\
0 & 0 & 0 & V_{4223}^{-1} & 0 & 0 & 0 & V_{4224}^{-1} & 0 & 0 & V_{4225}^{-1} & 0 \\
V_{4311}^{-1} & V_{4322}^{-1} & V_{4333}^{-1} & 0 & 0 & 0 & 0 & 0 & V_{4334}^{-1} & 0 & 0 & V_{4335}^{-1} \\
0 & 0 & 0 & 0 & V_{5113}^{-1} & 0 & V_{5114}^{-1} & 0 & 0 & V_{5115}^{-1} & 0 & 0 \\
0 & 0 & 0 & V_{5223}^{-1} & 0 & 0 & 0 & V_{5113}^{-1} & 0 & 0 & V_{5225}^{-1} & 0 \\
V_{5311}^{-1} & V_{5322}^{-1} & V_{5333}^{-1} & 0 & 0 & 0 & 0 & 0 & V_{5334}^{-1} & 0 & 0 & V_{5335}^{-1}
\end{bmatrix}.
\tag{17}
$$

Completely explicit expressions for these non-zero entries of the above matrix have been accomplished in this work; however, the results are too lengthy to be listed here but are tabulated in Appendix B for continuous fibers. Once the inverse of the tensor V_{iJAb} is obtained, it can be used with Eqn 16 and (B1)-(B4) to obtain closed-form solutions of the magnetoelectric coupling coefficients for continuous fibers. After some straightforward but tedious algebratic manipulations, the closed-form solutions are written out compactly as follows.

$$
\begin{aligned}
\bar{\lambda}_{11} &= \bar{\lambda}_{22} = \frac{-4f(1-f)e_{15}^{1}q_{15}^{0}\kappa_{11}^{0}\Gamma}{Y_0 + Y_1 f + Y_2 f^2 + Y_3 f^3}, \\
\bar{\lambda}_{33} &= \frac{2f(1-f)e_{31}^{1}q_{31}^{0}}{(1-f)(C_{12}^{0} - C_{12}^{1} - C_{11}^{1}) - (1+f)C_{11}^{0}},
\end{aligned}
\tag{18}
$$

where

$$
\begin{aligned}
Y_0 &= {e_{15}^{1}}^{2}(\Gamma_{11}^{0} + \Gamma_{11}^{1}) + (\kappa_{11}^{0} + \kappa_{11}^{1})\{({q_{15}^{0}}^{2} + (C_{44}^{0} + C_{44}^{1})(\Gamma_{11}^{0} + \Gamma_{11}^{1})\}, \\
Y_1 &= -{e_{15}^{1}}^{2}(\Gamma_{11}^{0} + 3\Gamma_{11}^{1}) + \kappa_{11}^{0}\{3{q_{15}^{0}}^{2} + C_{44}^{1}(\Gamma_{11}^{0} - \Gamma_{11}^{1}) + C_{44}^{0}(3\Gamma_{11}^{0} + \Gamma_{11}^{1})\} \\
&\quad + \kappa_{11}^{1}\{{q_{15}^{0}}^{2} + C_{44}^{0}(\Gamma_{11}^{0} - \Gamma_{11}^{1}) - C_{44}^{1}(\Gamma_{11}^{0} + 3\Gamma_{11}^{1})\}, \\
Y_2 &= -{e_{15}^{1}}^{2}(\Gamma_{11}^{0} - 3\Gamma_{11}^{1}) + \kappa_{11}^{0}\{3{q_{15}^{0}}^{2} + 3C_{44}^{0}\Gamma_{11}^{0} - C_{44}^{1}\Gamma_{11}^{0} - \Gamma_{11}^{1} \\
&\quad + (C_{44}^{0} + C_{44}^{1})\Gamma_{11}^{1}\} - \kappa_{11}^{1}\{{q_{15}^{0}}^{2} + C_{44}^{1}(\Gamma_{11}^{0} - 3\Gamma_{11}^{1}) + C_{44}^{0}(\Gamma_{11}^{0} + \Gamma_{11}^{1})\}, \\
Y_3 &= (\kappa_{11}^{0} - \kappa_{11}^{1}) + \{{q_{15}^{0}}^{2} + (C_{44}^{0} - C_{44}^{1})(\Gamma_{11}^{0} - \Gamma_{11}^{1})\} + {e_{15}^{1}}^{2}(\Gamma_{11}^{0} - \Gamma_{11}^{1}).
\end{aligned}
\tag{19}
$$

It is seen from Eqn 18 that the magnetoelectric coupling coefficients are a function of phase properties, volume fraction, and inclusion shape. It is also of interest to examine the behavior of the present model for the two-phase piezoelectric-piezomagnetic composite in the low (dilute) and high concentration limits. As $f \to 0$ and $f \to 1$, the magnetoelectric coupling

coefficients vanish. This verifies that the magnetoelectric coupling coefficients are absent in each constituent.

With Eqn 18 in hand, it is easy to obtain closed form for optimized f which results in highest magnetoelectric coupling effect. By differentiating $\bar{\lambda}_{33}$ with respect to f and then setting zero, the optimized f is achieved as

$$f = \frac{\sqrt{C_{11}^0 - C_{12}^0 + C_{11}^1 + C_{12}^1}}{\sqrt{2C_{11}^0} + \sqrt{C_{11}^0 - C_{12}^0 + C_{11}^1 + C_{12}^1}} . \tag{20}$$

It is noted that only optimized f for $\bar{\lambda}_{33}$ is necessary because the magnitude of $\bar{\lambda}_{33}$ is much larger than that of $\bar{\lambda}_{11}$. This will be shown in Fig. 1.

NUMERICAL RESULTS AND DISCUSSIONS

Numerical examinations have been conducted for a $BaTiO_3$-$CoFe_2O_4$ composite. The reinforcement is the piezoelectric $BaTiO_3$ continuous fibers and matrix is the piezomagnetic $CoFe_2O_4$. The material constants are given as follows (see Huang and Kuo [6]):

$BaTiO_3$ continuous fibers:

$$\begin{aligned}
&C_{11}^1 = 166 \text{ GPa},\ C_{33}^1 = 162 \text{ GPa},\ C_{44}^1 = 43 \text{ GPa},\\
&C_{12}^1 = 77 \text{ GPa},\ C_{13}^1 = 78 \text{ GPa},\\
&e_{11}^1 = -4.4 \text{ C/m}^2,\ e_{33}^1 = 18.6 \text{ C/m}^2,\ e_{15}^1 = 11.6 \text{ C/m}^2,\\
&\kappa_{11}^1 = 11.2\times10^{-9} \text{ C}^2/\text{Nm}^2,\ \kappa_{33}^1 = 12.6\times10^{-9} \text{ C}^2/\text{Nm}^2,\\
&\Gamma_{11}^1 = 5.0\times10^{-6} \text{ Ns}^2/\text{C}^2,\ \Gamma_{11}^1 = 10.0\times10^{-6} \text{ Ns}^2/\text{C}^2.
\end{aligned} \tag{21}$$

$CoFe_2O_4$ matrix:

$$\begin{aligned}
&C_{11}^0 = 286 \text{ GPa},\ C_{33}^0 = 269.5 \text{ GPa},\ C_{44}^0 = 45.3 \text{ GPa},\\
&C_{12}^0 = 173.0 \text{ GPa},\ C_{13}^0 = 170.5 \text{ GPa},\\
&q_{31}^0 = 580.3 \text{ N/Am},\ q_{33}^0 = 699.7 \text{ N/Am},\ q_{15}^0 = 550 \text{ N/Am},\\
&\kappa_{11}^0 = 0.08\times10^{-9} \text{ C}^2/\text{Nm}^2,\ \kappa_{33}^0 = 0.093\times10^{-9} \text{ C}^2/\text{Nm}^2,\\
&\Gamma_{11}^0 = -590\times10^{-6} \text{ Ns}^2/\text{C}^2,\ \Gamma_{33}^0 = 157\times10^{-6} \text{ Ns}^2/\text{C}^2.
\end{aligned} \tag{22}$$

The results for the magnetoelectric coupling $\bar{\lambda}_{11}$ and $\bar{\lambda}_{33}$ against f are plotted in Fig. 1. Both coefficients are found to vanish at f = 0(pure matrix) and f = 1 (pure fiber) in which cases the materials become monolithic. However, the magnitude of $\bar{\lambda}_{33}$ is much larger than that of $\bar{\lambda}_{11}$. The most significant coupling effect is found to occur at f =0.46 for $\bar{\lambda}_{33} = 2.4\times10^{-9}$ Ns/VC. It is found in Eqn 20 that the optimized fiber volume fraction is only function of elastic moduli of fiber and matrix in the transverse direction with respect to fiber axis, but not related to piezoelectric cofficient e_{nij} or piezomagnetic cofficient q_{nij}. The slight excess of $BaTiO_3$ in composition but less than its optimized value results in a much higher magnetoelectric coupling effect. The effect may relate to change of microstructure (Boomgaard et al., [10]; Van Run et

al. [11]). The magnetoelectric couoling effect is due to the mechanical coupling of the piezomagnetic spinel cobalt ferrite-cobalt titanate and the piezoelectric perovskite barium titanate at the interphase region. A change in the magnetization arises in the piezomagnetric matrix due to elastic strain generated by the piezoelectric continuous fibers under the influence of an applied electric field. The continuous fiber reinforced composite has a strong mechanical coupling between two phases, transferring elastic strain without appreciable losses, so that the composite provides a necessary condition for above coupling effect. The strong mechanical coupling infers that the optimized f is only function of the mechanical property C_{ijmn}. The dependence of the magnetoelectric coupling $\bar{\lambda}_{33}$ on special elastic moduli may be owing to the growth direction of above phases. An additional advantage of the continuous fiber composite is the well-defined crystal orientation of the two phases with respect to the fiber direction and with respect to each other. One can expect the crystallographic alignment of all crystallites constituting fibers to give optimum coupling effect of the composite. When f is higher than its optimized value, the spinel dendrites in the composite reduce due to the denser packing of $BaTiO_3$ fibers. Therefore, the magnetoelectric coupling effect decreases due to less mechanical coupling between spinel and perovskite. The contribution of the result is that it is easier to achieve optimized fiber volume fractions for maximum magnetoelectric coupling effects in various composites without measuring e_{nij} and q_{nij}

CONCLUSIONS

The Mori-Tanaka mean-field theory and the inclusion method have been extended to derive the explicit formulae for magnetoelectric coupling effect in piezoelectric/piezomagnetic composites. By differentiating the formulae with respect to fiber volume fraction and setting zero, the optimized fiber volume fraction for maximum magnetoelectric coupling effects in the composite can be easily obtained. For the $BaTiO_3$-$CoFe_2O_4$ composite, the maximum value for the most significant magnetoelectric coupling $\bar{\lambda}_{33}$ was found to be 2.4×10^{-9} N s/VC as fiber volume fraction was 0.46, while the maximum value of $\bar{\lambda}_{11}$ was $0.007\text{x}10^{-9}$ N s/VC which is much smaller compared to that of $\bar{\lambda}_{33}$. Based on the results, optimized fiber volume fractions can be obtained for various composite materials with maximum magnetoelectric coupling effect.

ACKNOWLEDGMENTS

Authors are grateful for financial support from National Science Council in Taiwan on this work under contracts NSC 87-2216-E-035-016 (JHH) and NSC 87-2218-E-006-013 (CKC).

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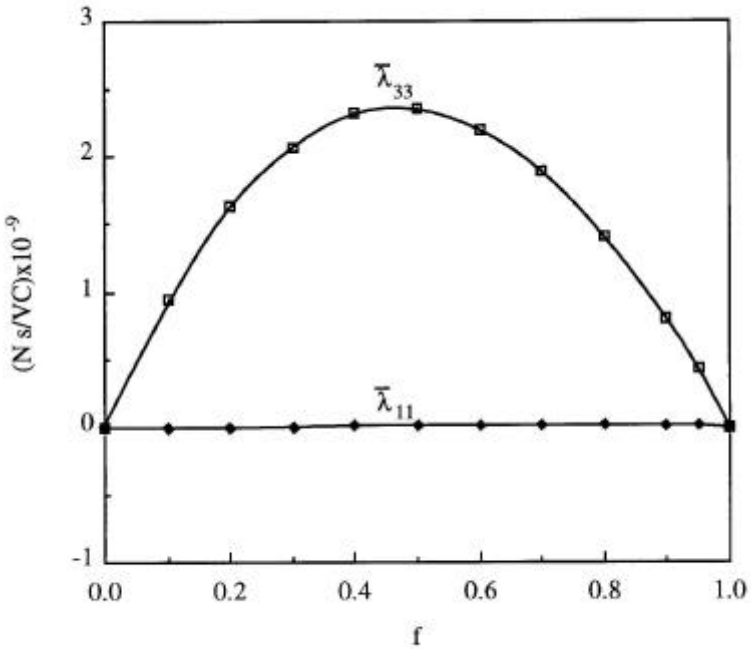

Fig. 1 : Effect of fiber volume fraction on $\bar{\lambda}_{33}$ and $\bar{\lambda}_{11}$.