# INFLUENCE OF THE THERMAL DEPENDENT MATRIX PROPERTIES ON THE RESIDUAL STRESS DISTRIBUTION IN A MODEL COMPOSITE

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**SUMMARY**: Usually, the resin has a significantly higher thermal elongation than the reinforcing fibres. This mismatch leads, during the curing process, to thermal residual stresses. It will be shown that values of thermal residual stresses in a fibre reinforced composite strongly depend on the stiffness behaviour of the resin. Among others the stiffness of the resin is a function of temperature and strain. The influence of the thermal dependent matrix stiffness was examined by a comparison of finite element analysis and photoelastic examination. For the implementation of the non-linear material behaviour into the FEA-Program the results of tensile tests and thermal mechanical analysis were fitted with suitable mathematical functions. The results of the FEA calculations with thermal dependent matrix properties are in good comparison with the results from the photoelastic analysis.

**KEYWORDS**: thermal dependent material properties, finite element analysis, photoelastic analysis, glass fibre, epoxy resin.

## INTRODUCTION

For the calculation of fibre reinforced composite properties, i.e. the strength and the stress distribution within the composite, it is necessary to determine the initial stress state after curing. The existences of thermal residual stresses after hot curing is well known, however, it must be taken into account that the material properties of an epoxy resin are functions of temperature and strain. In the present work a finite element analysis (FEA) was performed (Fig. 1a) and verified by a transverse fibre test. The non-linear material behaviour of the matrix was implemented in an FEA-Program for the calculation of the residual stress distribution.

The actual stress distribution around a single fibre can be estimated by quantitative photo elastic analysis, similar to the analysis during a fibre fragmentation test [1]. A specimen for this test is schematically shown in Fig. 1b.

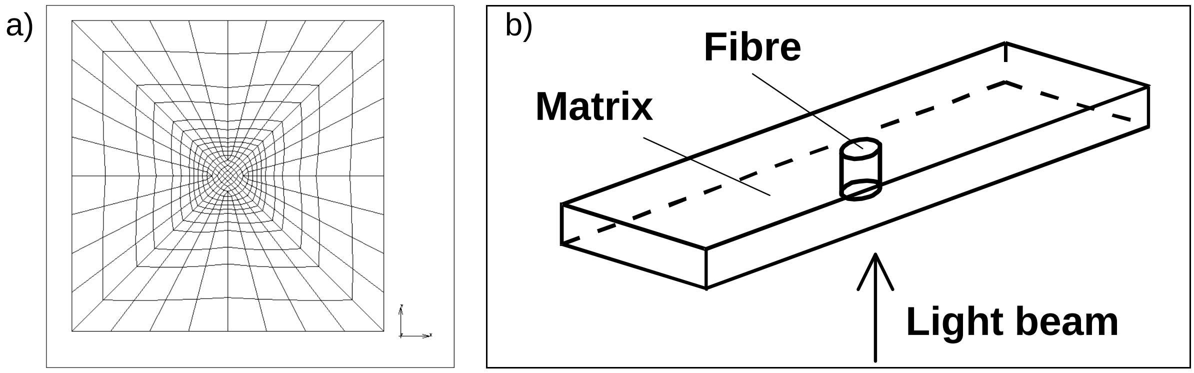


Fig. 1: a) FEA-mesh for the calculation of thermal residual stresses;
b) Single fibre E-glass/epoxy model composite (schematically).

It is necessary to determine the photo elastic constant to correlate the observed fringe pattern with the local stress distribution. The mean shear stress can be calculated from the fringe pattern using Eqn 1.

$$\tau = \frac{S}{h} \cdot \delta \tag{1}$$

## EXPERIMENTAL

For the matrix a hot cured Ciba-Geigy LY556®/HY932® epoxy resin, flexibilised with JEFFAMINE D2000® (Huntsman Corp.) was used. The amount of flexibiliser causes a marked non linearity of the stress-strain behaviour. The curing cycle was 3h at 80°C and 8h at 120 °C. The E-glass fibres (diameter 80µm) uncoated and coated with different sizing were supplied by IPF-Dresden (Germany). The material properties are summarised in Table 1.

Table 1: Material properties.

| | Elastic Modulus E [MPa] | Shear Modulus G [MPa] | Poissons Ratio $\nu$ [1] | Thermal Expansion Coefficient $\alpha$ [mm/m K] | Photo Elastic Constant S [MPa*mm] |
|---|---|---|---|---|---|
| E-Glass Fibre | 75000 | 30240 | 0.24 | $1*10^{-6}$ | -- |
| Epoxy Resin M16 | 2250 | 827 | 0.36 | $8*10^{-5}$ | 6.6 |

The thermal dependent matrix stiffness was determined by thermal dynamic elastic analysis (Rehometrics RDA II). The resulting shear modulus can be converted into the elastic modulus using Eqn 2.

$$E = G \cdot 2 \cdot (1 + \nu) \tag{2}$$

The strain dependence of the matrix stiffness was determined by tensile tests with dog bone specimens and a constant strain velocity of 1 mm/min (Zwick 1475).

The non-linear material behaviour was implemented using a FORTRAN subroutine. Therefore it is necessary to find a suitable fitting function for the description of the measured values. The following Eqn 3 depicts the non-linear stress-strain relation of the epoxy resin [2].

$$\sigma(\varepsilon) = A \cdot \tanh(B \cdot \varepsilon) \tag{3}$$

The first derivative of the stress σ with respect to the strain ε leads to the strain dependent tangent modulus $E_{Tan}(\varepsilon)$ (Eqn 4).

$$E_{Tan}(\varepsilon) = \frac{d\sigma}{d\varepsilon} = \frac{A \cdot B}{\cosh^2(B \cdot \varepsilon)} \tag{4}$$

A similar expression can be found for the temperature dependent tangent modulus $E_{Tan}(T)$ (Eqn 5).

$$E_{Tan}(T) = \frac{C}{\cosh^2(D \cdot T + E)} \tag{5}$$

If the strain dependent tangent modulus $E_{Tan}(\varepsilon)$ is related to the spontaneous elastic modulus $E_{Tan}(\varepsilon \approx 0)$ (Eqn 6),

$$\frac{E_{Tan}(\varepsilon)}{E_{Tan}(0)} = \frac{\dfrac{A \cdot B}{\cosh^2(B \cdot \varepsilon)}}{\dfrac{A \cdot B}{\cosh^2(0)}} = \frac{1}{\cosh^2(B \cdot \varepsilon)} \tag{6}$$

the strain and temperature dependent tangent modulus can be written as a combination of Eqn 5 and Eqn 6:

$$E_{Tan}(T, \varepsilon) = \frac{C}{\cosh^2(B \cdot \varepsilon) \cdot \cosh^2(D \cdot T + E)} \tag{7}$$

The values calculated with Eqn 7 are graphically shown in Fig. 2.

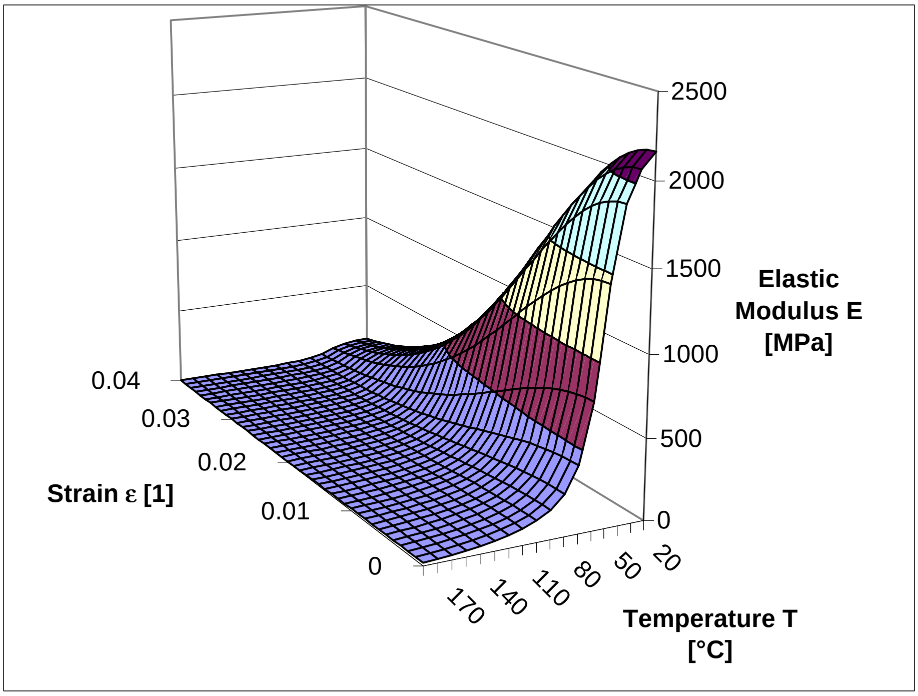


Fig. 2: Tangent modulus as a function of temperature and strain.

## RESULTS

The fit of a typical stress-strain curve shows a good correlation of the chosen function and the measured values (Fig. 3) [2].

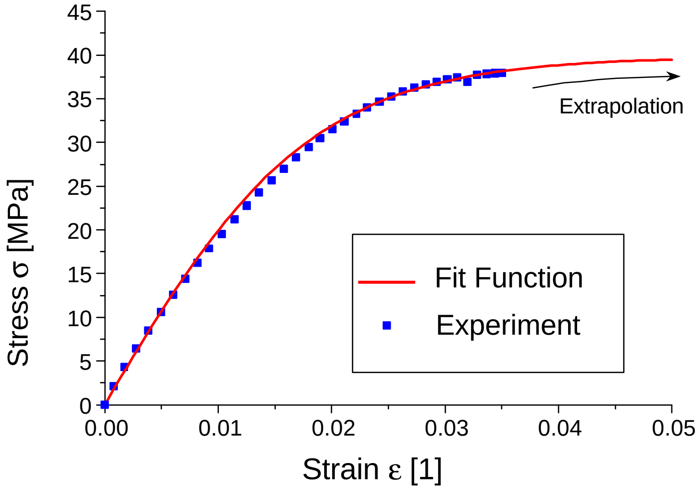


Fig. 3: Stress σ versus strain ε , including the fit function from Eqn. 2.

In case of a temperature dependent resin stiffness the fitting function is in good agreement with the measured values too, however, for relatively high temperatures the calculated values are slightly lower (Fig. 4).

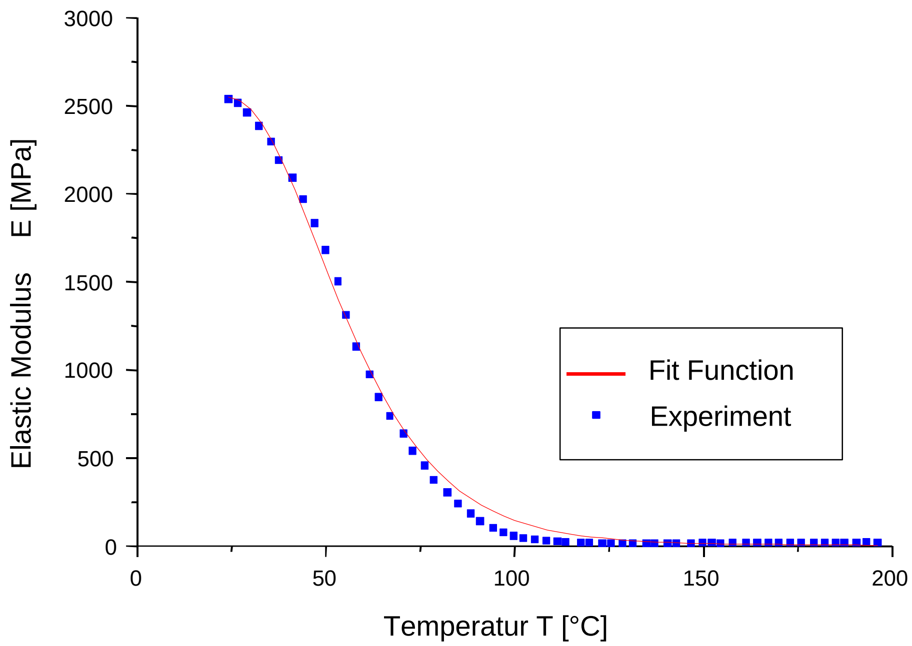


Fig. 4: Elastic modulus E versus temperature T, including the fit function from Eqn. 5.

The variables of the fit functions (Eqns. 3 and 5) are summarised in Table 2.

Table 2: Variables of the fit functions Eqns. 3 and 5.

| A [MPa] | B [1] | C [MPa] | D [1/K] | E [1] |
|---|---|---|---|---|
| 39.8 | 55.2 | 2546 | 0.028 | -0.679 |

The quantitative photo elastic analysis was chosen for the evaluation of the numerically calculated results. Therefore, the determination of the photo elastic constant is necessary to correlate the observed fringe pattern with the local stress distribution. The photo elastic constant was determined to S = 6.6 MPa*mm using the test equipment shown in Fig. 5.

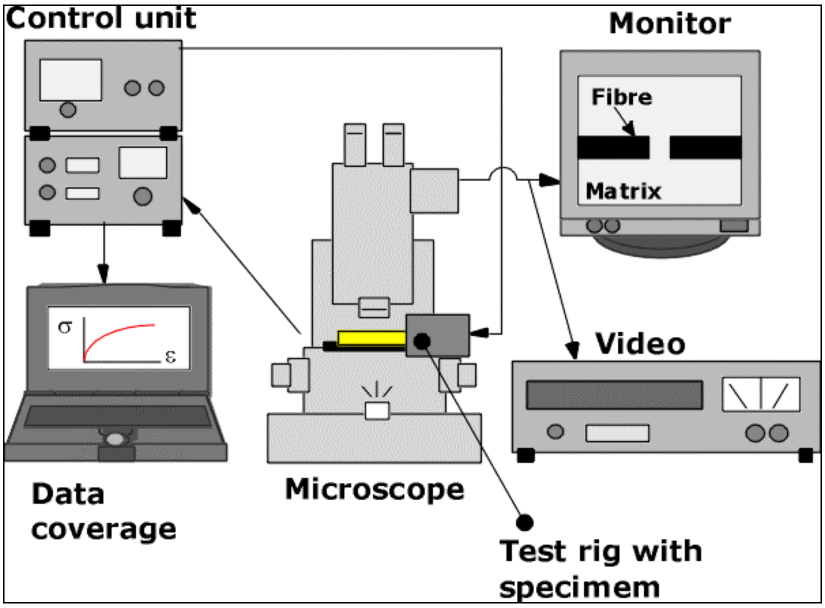


Fig. 5: Test equipment for the photo elastic analysis (schematically).

The results of the FEA-calculation will be presented for:

a) a resin with constant elastic modulus,
b) a resin with a temperature dependent elastic modulus and
c) a resin with a temperature and strain dependent elastic modulus.

The result of the FEA-model with constant elastic modulus shows a linear increase in stress with decreasing temperature; finally at room temperature a shear stress of 26 MPa (Fig. 6) occurs.

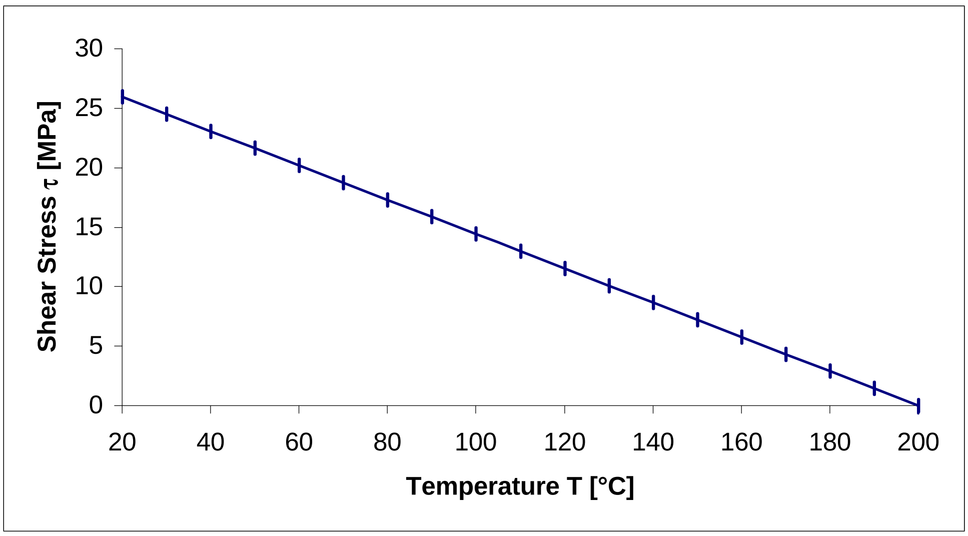


Fig. 6: Shear stress τ versus temperature T, constant elastic modulus.

In the case of a calculation with thermal dependent matrix properties the residual stresses are significantly lower. The stress increases progressively with decreasing temperature and at room temperature a shear stress of 5.76 MPa (Fig. 7) occurs.

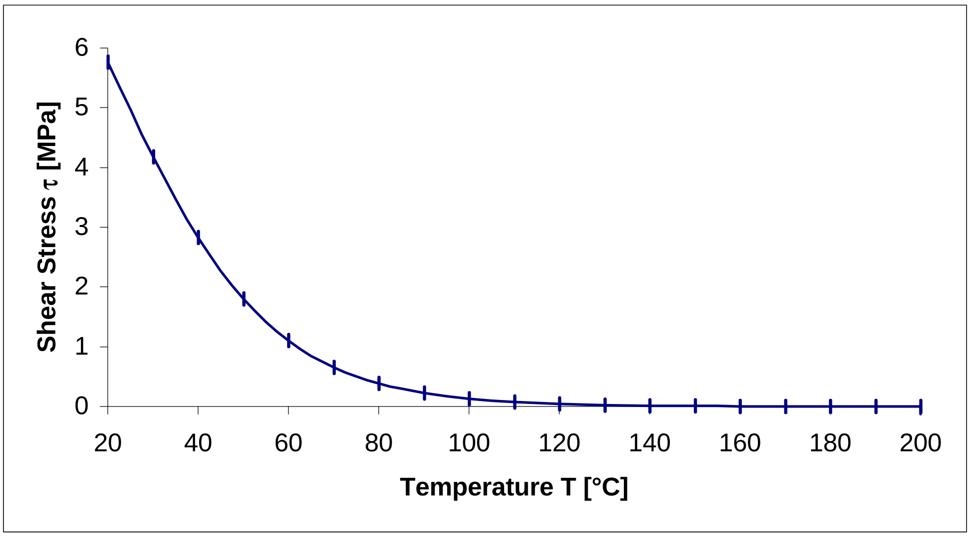


Fig. 7: Shear stress τ versus temperature T, temperature dependent elastic modulus.

The calculation with the temperature and strain dependent elastic modulus shows slightly lower values of the compression stress than the previous calculations. The maximum thermal residual stress amounts to 4.03 MPa (Fig. 8).

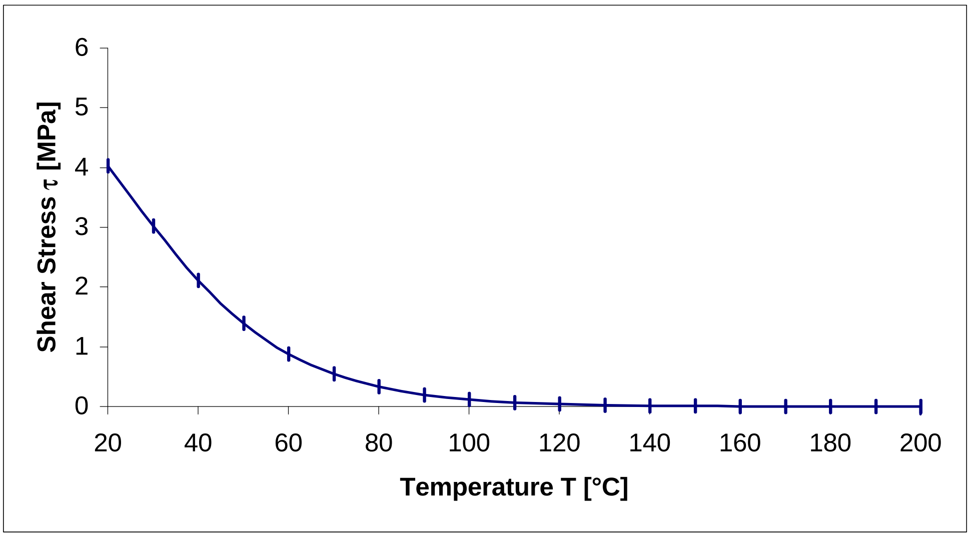


Fig. 8: Shear stress τ versus temperature T, temperature and strain dependent elastic modulus.

The resulting values of approximately 4-6 MPa for the calculation with temperature- and temperature-strain dependent elastic modulus are in good agreement with the experimental results (Fig. 9).

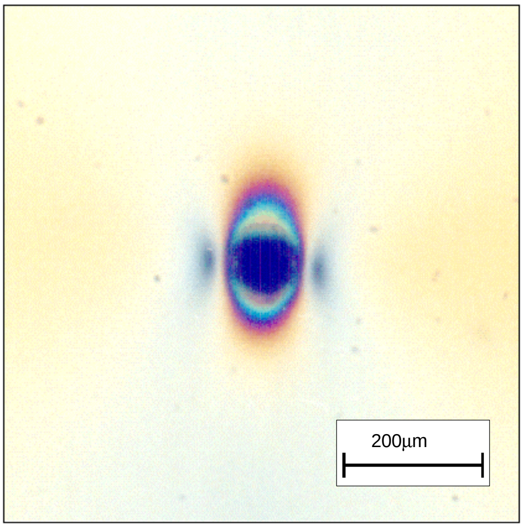


Fig. 9: Photo elastic fringe pattern surrounding a single glass fibre.

The existence of one single fringe pattern surrounding the fibre correlates with a main shear stress value off approximately 6 MPa [3]. The asymmetric shape of the fringe pattern is caused by the specimen dimensions, i.e. the specimens are much longer than wide.

## CONCLUSIONS

The comparison between photo elastic analysis and FEA shows the need of the implementation of thermal and strain dependent material properties to receive realistic results in the calculation of thermal residual stresses in fibre reinforced composites. A calculation with a thermal independent elastic modulus leads to unrealistically high values of the shear stress $\tau = 26$ MPa. The effect of the strain dependent matrix stiffness is relatively low in comparison to the influence of thermal matrix properties. For a more exact determination of thermal residual stresses the temperature dependence of the thermal expansion coefficient has to be taken into account.

## ACKNOWLEDGEMENTS

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