USE OF A LINEAR VISCOELASTIC CONSTITUTIVE LAW TO MODEL THE TIME DEPENDENT BEHAVIOUR OF ORTHOTROPIC GLASSFIBRE REINFORCED PLASTIC

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SUMMARY: Polymeric matrix composites show a strong time-dependent behaviour. This has also been observed in long-term tests conducted at our department. One of the most powerful tools to model this time dependent behaviour is the theory of viscoelasticity. Orthotropic isothermic linear behaviour is assumed for the material under study. A three-dimensional constitutive law formulated by superposition integrals is implemented in the FEM-program ABAQUS. The ability of the implemented numerical algorithm to predict the response of a long thick-walled cylindrically anisotropic cylinder that is subjected to internal pressure is shown (a problem for which the analytical solution is known). The main future goal is to combine the study of time dependent behaviour with the analysis of the lifetime durability by means of damage mechanics. The analyst will be provided with a versatile tool with which it is possible to model the time-dependent behaviour of a wide range of materials.

KEYWORDS: viscoelasticity, constitutive model, Finite Element Method, anisotropy, creep, damage mechanics, life-time analysis

INTRODUCTION

One of the aims of our co-operation with an industrial company is to investigate the time dependent behaviour of a centrifugally cast filled glass fibre reinforced unsaturated polyester resin pipe. The pipe consists of several different layers, which exhibit either isotropic or orthotropic behaviour. The fibres have a length of 50 [mm] and calcium carbonate is used as filler.

Polymeric matrix composites show a strong time-dependent behaviour. This has also been observed in long-term tests conducted at our department. For example we applied a constant load at the top of a pipe. This load was maintained constant for 1127 hours. The deformation of the vertical diameter increased in this time by the half of the short-term deformation.

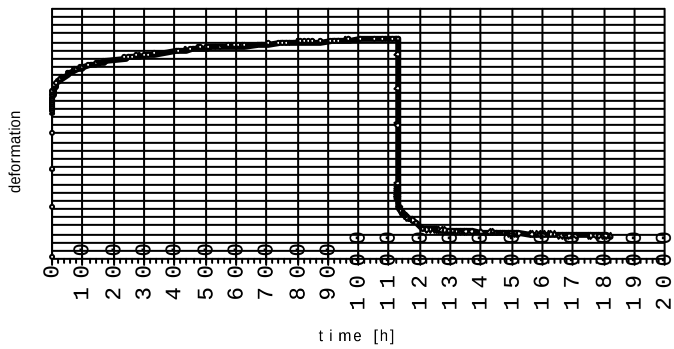

Fig. 1: deformation of the vertical diameter vs. time

IMPLEMENTATION OF AN ORTHOTROPIC ISOTHERMIC LINEAR VISCOELASTIC CONSTITUTIVE LAW IN A FINITE ELEMENT FORMULATION

One of the most powerful tools to model this time-dependent behaviour is the theory of viscoelasticity. For this material linear viscoelastic behaviour is assumed. Different theoretical approaches for linear viscoelasticity are well known: spring-dashpot-models, mathematical models, superposition integrals. For computational strategies, the superposition integrals offer undeniable advantages over that of discrete state variables. The integral formulation is based on the Boltzmann superposition principle [1]. We derive the superposition integral by considering an arbitrary strain history as consisting of a sequence of steps $d\varepsilon$ at moments τ:

$$\sigma_{ij}(t) = \int_0^t C_{ijkl}(t-\tau)\frac{\partial \varepsilon_{kl}}{\partial \tau}\, d\tau \qquad (1)$$

The term $C_{ijkl}(t)$ represents the fourth order tensor of the relaxation moduli.

The constitutive law for linear viscoelasticity in Eq.1 is used to develop a numerical algorithm for orthotropic media which is implemented in the commercial FE-code of ABAQUS. The constitutive equations are transformed into an incremental algebraic form. This incrementation is accomplished in closed form and results in a recursive relationship. $C_{ijkl}(t)$ can be determined by relaxation tests and the curves can be fitted in the best way by Dirichlet-Prony series corresponding to a generalised Maxwell model:

$$C_{ijkl}(t) = C_{ijkl\infty} + \sum_{m=1}^{M_{ijkl}} C_{ijklm}\cdot e^{\frac{-t}{\rho_{ijklm}}} \qquad \rho_{ijklm} = \frac{\eta_{ijklm}}{C_{ijklm}} \qquad (2)$$

C_{ijklm} spring constants
ρ_{ijklm} relaxation times
η_{ijklm} dashpot coefficients

The use of Dirichlet-Prony series leads to the recursive relationship:

$$s_{ij}(t) = C_{ijkl_0} \cdot \left(e_{kl} - \sum_{m=1}^{M_{ijkl}} \frac{C_{ijkl_m}}{C_{ijkl_m}} \cdot e_{kl_m}^{ijkl} \right) \qquad (3)$$

short-term long-term

VERIFICATION

We investigate the ability of the implemented numerical algorithm to predict the response of a long thick-walled cylindrically anisotropic cylinder that is subjected to a internal pressure p=P*H(t) with H(t) as the Heaviside step function. This problem has previously been investigated by Schapery [3] and Zocher [4] and an accepted analytical solution is known . In Schapery's analysis, the response of the cylinder was predicted by three different methods: (1) correspondence principle, (2) quasi-elastic method, and (3) collocation. Here we duplicate the first two analyses of Schapery and compare the results to FE prediction (Fig.3). The derivation of a solution by correspondence principle is based on a corresponding elastic solution for general orthotropy by Lekhnitskii [2]. Following Schapery we concern ourselves only with the prediction of the hoop stress, σ_{tang} at the inner wall of the cylinder. The results are satisfying and the FE-prediction fits the analytical solutions very well. A hypothetical material with different time dependent relaxation moduli in radial and circumferential direction was used to calculate the $C_{ijkl}(t)$ which were fitted in form of Dirichlet-Prony series (Fig.2).

$$E_{rad} = E_e \cdot \left[1 + 100 \cdot (t)^{-0.5}\right] \qquad (4)$$

$$E_{\tan g} = E_e \cdot \left[1 + 100 \cdot (t)^{-0.1}\right] \qquad (5)$$

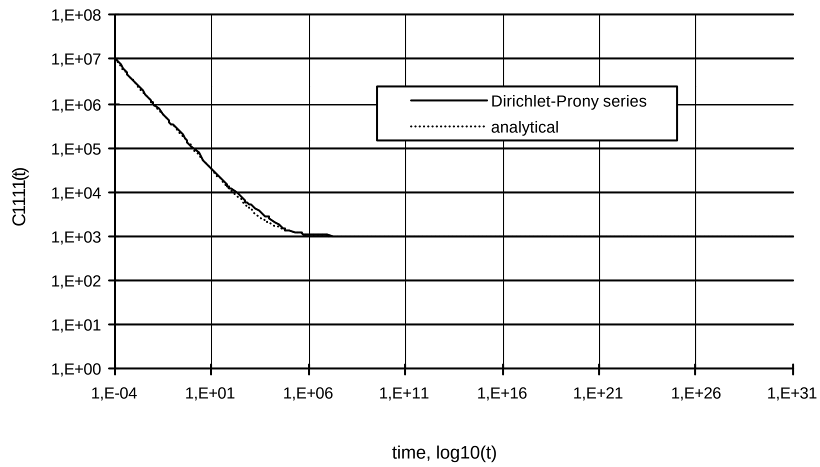

Fig. 2: $C_{1111}(t)$

The FE mesh used in the analysis consisted of 270 plane strain elements (8-node biquadratic). The FE mesh was not equidistant, it was contracted by a factor of 0.6 from the outer to the inner wall. The stress was calculated in the integration points of the elements. Time steps of variable length were used with the magnitude of the time step slowly increasing as time progressed. This variability in time step size was not accomplished adaptively, but assigned in the input file. A formal convergence study of the sensitivity of time step size has not been conducted at this moment.

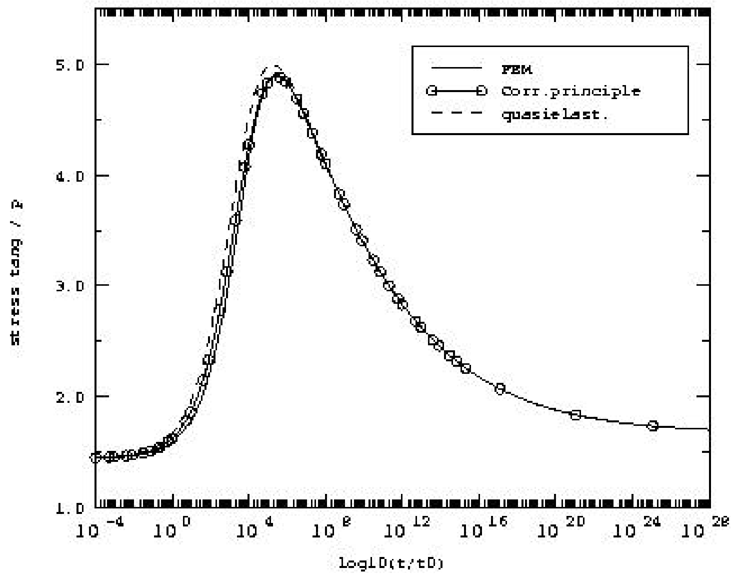

Fig. 3: tangential stress at the inner wall versus time

FUTURE GOALS

The main future goal is to combine the study of time dependent behaviour with the study of the onset and evolution of damage. Based on the herein formulated constitutive law (Eqn 1) a three dimensional coupled viscoelasticity/damage constitutive law will be implemented in the FEM-program ABAQUS. This law should be capable of modelling nonlinear viscoelastic behaviour and the lifetime durability by means of damage mechanics. [5] [6] [7]

CONCLUSION

A linear viscoelastic constitutive law for a orthotropic material has been implemented in the FE-program Abaqus. This development provides the analyst with a versatile tool with which he can model the time dependent behaviour of a wide range of materials.

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